Robust production planning and control for multi-stage systems with flexible final assembly lines

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1. Introduction

Dynamic lot size model with returns and remanufacturing

(1) Multiple product
(2) Operator is needed
(3) Capacity restriction
(4) When and how much to manufacture product for each stage (Machinery // Assembly lines)
(5) When and how many operators are needed for each stage
(6) 2 kinds of setup occur (To produce module // To assemble product)
(7) Uncertain parameters
2. Problem Description

- **Objective function**
  1. Minimize the total cost (deviation[early delivery and holding], setup, personnel costs)
  2. Minimize the total cost (human labor and total inventory costs)

- **Decision variables**
  1. Order selection ($X_{it}: \{0,1\}$)
  2. Production quantity for product $p$ belongs to the order $i$ ($q_{pt}$)\// one’s setup ($y_{pt} : \{0,1\}$)
  3. Number of operators in shift $t$
  4. Production quantity for module $m$ in time $\pi$ on machine $j$\// setup
  5. Inventory level of module $m$ in time $\pi$
  6. Assignment of operator to machine $j$ in time $\pi$
  7. Production quantity of module $m$ by operator $o$ in time $\pi$ on machine $j$

- **Assumptions**
  1. Initial inventory
  2. Backlogging is allowed
  3. All parameters are to be nonnegative
  4. Lead time

- **Approach (Next page)**
2. Problem Description

Overall explanation for approach

Rework rates, machine downtimes, operator movement, capacity control-related effects
2. Problem Description

Notation

Sets

\( N \)  
set of production orders

\( P \)  
set of products

\( T \)  
set of working shifts (macro-periods)

\( \Pi \)  
set of micro-time periods

\( M \)  
set of product modules

\( J \)  
set of machines

\( O \)  
set of operators in the machinery

Parameters

\( p_i \)  
product of order \( i \)

\( v_i \)  
volume of order \( i \) [pcs.]

\( r_i^d \)  
due date of order \( i \) [shift]

\( r_i^p \)  
the total manual norm cycle time of product \( p \)

\( r_m^s \)  
machine setup time of module \( m \) [min]

\( r_m^o \)  
manual operation time of module \( m \) [min]

\( r_m^c \)  
machining time of module \( m \) [min]

\( r_i^w \)  
duration of a working shift (macro-period length) [min]

\( r_i^\pi \)  
length of a micro-period [min]

\( k \)  
ratio of the macro- and micro-periods’ length: \( t^w = k r^\pi \ k \in \mathbb{Z}^+ \)
## 2. Problem Description

### Notation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_m^l$</td>
<td>lead time of module $m$ in the shared resources segment $[t^\pi]$</td>
</tr>
<tr>
<td>$c_i^h$</td>
<td>inventory holding cost (product or module) [cost/part/shift]</td>
</tr>
<tr>
<td>$c_i^l$</td>
<td>late delivery cost [cost/product/shift]</td>
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<tr>
<td>$c_{ii}$</td>
<td>deviation cost of order $i$ in shift $t$</td>
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<tr>
<td>$c_s$</td>
<td>cost of a setup</td>
</tr>
<tr>
<td>$c_o$</td>
<td>cost of an operator per shift</td>
</tr>
<tr>
<td>$w_{\max}$</td>
<td>the max. number of operators working in the same shift</td>
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</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{i \pi t}$</td>
<td>production of order $i$ in shift $t$ (binary)</td>
</tr>
<tr>
<td>$y_{p \pi t}$</td>
<td>production of product $p$ in shift $t$ (binary indicator variable)</td>
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<tr>
<td>$q_{p \pi t}$</td>
<td>produced volume from product $p$ in shift $t$ (integer)</td>
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<tr>
<td>$w_t$</td>
<td>number of operators working in shift $t$ (integer)</td>
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<td>$z_{m \pi \pi j}$</td>
<td>volume of module $m$ machined in time $\pi$ on machine $j$</td>
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<tr>
<td>$Y_{m \pi \pi j}$</td>
<td>production of module $m$ in time $\pi$ on machine $j$ (binary indicator variable)</td>
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<tr>
<td>$z_{m \pi \pi j}$</td>
<td>setup on machine $j$ for module $m$ in time $\pi$ (indicator variable)</td>
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<tr>
<td>$h_{m \pi}$</td>
<td>inventory level of module $m$ in time $\pi$</td>
</tr>
<tr>
<td>$r_{o \pi j}$</td>
<td>assignment of operator $o$ and machine $j$ in time $\pi$</td>
</tr>
<tr>
<td>$\omega_{m \pi o}$</td>
<td>volume of module $m$ machine by operator $o$ in time $\pi$ on machine $j$</td>
</tr>
</tbody>
</table>
2. Problem Description

Formulation

\[
\text{minimize } \sum_{i \in N, t \in T} c_{it} x_{it} + c^s \sum_{p \in P, t \in T} y_{pt} + c^o \sum_{t \in T} w_t \tag{2}
\]

subject to

\[
\sum_{t \in T} x_{it} = 1 \quad \forall i \in N \tag{3}
\]

\[
x_{it} \leq y_{pt} \quad \forall t, p = p_i \tag{4}
\]

\[
q_{pt} = \sum_{i \in N} x_{it} v_i \quad \forall t \in T, p \in P \tag{5}
\]

\[
t^w w_t \leq Q \left( \bar{q}_t \right) \quad \forall t \in T \tag{6}
\]

\[
w_t \leq w_{\max} \quad \forall t \in T \tag{7}
\]

\[
x_{it} \in \{0, 1\}, \quad y_{pt} \in \{0, 1\}, \quad w_t \in \mathbb{Z}^+ \tag{8}
\]

\[
Q \left( \bar{q}_t \right) = \beta_0 + \beta_1 w_t + \sum_{p \in P} \beta_{p+1} q_{pt}
\]

Deviation cost

\[
c_{it} = \begin{cases} 
    c^h v_i (t_i^d - t) & \text{if } t < t_i^d \\
    c^l v_i (t - t_i^d) & \text{otherwise}
\end{cases}
\]

Order assignment

Order release = product setup

Product volume calculation

Labor capacity restriction : required # of workers

Required # of workers <= max # of workers
2. Problem Description

Formulation

\[
\begin{align*}
\text{minimize} & \quad c_0 \sum_{k} \sum_{o \in O} \sum_{j \in J} \sum_{\pi \in \Pi} r_{o,j,\pi} + \sum_{m \in M} \sum_{\pi \in \Pi} h_{m,\pi} \\
\text{subject to} & \quad h_{m,\pi} \geq q_{m,\pi} \\
& \quad h_{m,\pi} = h_{m,\pi-1} + \sum_{j \in J} z_{m,\pi-\left[i_m\right]_j} - d_{m,\pi} \\
& \quad \forall m \in M, \pi \in \Pi, t \in T, \pi = kt \\
& \quad \gamma_{m,\pi j} \leq z_{m,\pi j} \\
& \quad \forall m \in M, \pi \in \Pi, j \in J \\
& \quad \zeta_{m,\pi j} \leq \Theta \gamma_{m,\pi j} \\
& \quad \forall m \in M, \pi \in \Pi, j \in J \\
& \quad \zeta_{m,\pi j} \geq \gamma_{m,\pi-1,j} \\
& \quad \forall m \in M, \pi \in \Pi, j \in J \\
& \quad \zeta_{m,\pi j} + \sum_{\mu \in M} (\zeta_{\mu,\pi j} - \gamma_{\mu,\pi-1,j}) \leq 1 - \gamma_{\mu,\pi-1,j} \\
& \quad \forall m \in M, \pi \in \Pi, j \in J \\
& \quad \zeta_{m,\pi j} \leq \gamma_{m,\pi j} \\
& \quad \forall m \in M, \pi \in \Pi, j \in J \\
& \quad \sum_{m \in M} (t^c_{m,z_{m,\pi j}} + t^s_{m,\zeta_{m,\pi j}}) \leq t^\pi \\
& \quad \forall j \in J, \pi \in \Pi \\
& \quad \Theta \geq t^\pi / \min\{t^c_m : m \in M\}
\end{align*}
\]
Formulation (continue)

\[ z_{m \pi j} = \sum_{o \in O} \omega_{m \pi jo} \quad \forall j \in J, \pi \in \Pi, m \in M \]  
(19)

\[ r_{oj\pi} \leq \sum_{m \in M} \omega_{m \pi jo} \quad \forall j \in J, \pi \in \Pi, o \in O \]  
(20)

\[ \sum_{m \in M} \omega_{m \pi jo} \leq \Lambda r_{oj\pi} \quad \forall j \in J, \pi \in \Pi, o \in O \]  
(21)

\[ \sum_{m \in M} \sum_{j \in J} t^{o}_{m} \omega_{m \pi jo} \leq t^{\pi} \quad \forall \pi \in \Pi, o \in O \]  
(22)

\[ \sum_{m \in M} \gamma_{m \pi j} \leq 1 \quad \forall j \in J, \pi \in \Pi \]  
(23)

\[ z_{m \pi j} \in \mathbb{Z}^{+} \quad \gamma_{m \pi j} \in \{0, 1\} \quad h_{m \pi} \in \mathbb{Z}^{+} \quad r_{oj\pi} \in \{0, 1\} \quad \omega_{m \pi jo} \in \mathbb{Z}^{+} \]  
(24)

Volume of module is produced by each operator

Assignment operator for each module

Labor capacity restriction

Labor capacity restriction 2: manual operation time cannot exceed micro period

One module type can be produce on each machine

\[ \Lambda \geq t^{\pi} / \min \{t^{o}_{m} : m \in M \} \]
3. Approach

Improvement initial solution

Production planning and capacity control of the final assembly lines

- Selection of the proper capacity control policies
  
  *Siemens Tecnomatix Plant Simulation*

- Prediction of the capacity requirements
  
  *R Studio*

Production and shift planning

*FICO Xpress : stopping condition with 6% of optimality gap*

Production and capacity planning of the pre-inventory process

*FICO Xpress : stopping condition with 6% of optimality gap*
4. Computational experiments

Environment
- Simens Tecnomatix Plant Simulation, R Studio, FICO Xpress (MILP solver)

DATA
- $T = \{24, 30, 36, 42\}$
- Amount of Order: Normal, High, Extreme
- Each order scenarios 10 different instances
- $J = 11$, $M = 14$
- $k = 4$, $t^\pi = 120$
- $120 \leq d_{max} \leq 200 \text{ ?}$

Benchmark: Average percentage and CPU time
- NTP: deterministic norm time-based planning (most ERP system)
- RPN: proposed method in this study: simulation and regression based robust planning method
- ITR: iterative form of simulation-based optimization: After simulation, refining required capacity
- RCT: counter part of NTP: lower and upper bound for cycle time
- RCO: extended version of RPN: regression model included formulation

Benchmark with 4 other methods: total 600 instances
## 4. Computational experiments

### Module parameters in the test case

<table>
<thead>
<tr>
<th>$M$</th>
<th>$t_{m}^1$</th>
<th>$t_{m}^c$</th>
<th>$t_{m}^o$</th>
<th>$h_{m0}$</th>
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</table>
4. Computational experiments

Results: the capacity prediction for a sample assembly line
### 5. Computational experiments

**Results: Benchmark of robust production planning methods**

| $|T|$ | Orders | Lateness [%] | Objective [%] | CPU Time [s] |
|----|--------|-------------|---------------|--------------|
|    |        | NTP  RPN  RCT  RCO  ITR | NTP  RPN  RCT  RCO  ITR | NTP  RPN  RCT  RCO  ITR |
| 24 | Normal | 98   73   79   82   83 | 95   97   100  98   95 | 8.8  8.8  14.0  14.1 209.2 |
|    | High   | 100  87   79   83   95 | 91   94   100  95   91 | 9.3  10.9 81.5  18.4  90.0 |
|    | Extreme| 99   92  79(8)  87   97 | 29   33  100(8) 35   30 | 11.2 131.1 68.3(8) 368.3 52.0 |
| 30 | Normal | 100  78   70   73   85 | 93   96   100  97   93 | 10.2 11.7 40.4  21.5 141.0 |
|    | High   | 98   93   81   88   98 | 84   89   100  91   84 | 10.9 15.5 370.3  81.0  25.0 |
|    | Extreme| 99   90  86(10) 85(2) 95 | 22   25  100(10) 29(2) 23 | 134.8 517.9 116.4(10) 764.7(2) 331.7 |
| 36 | Normal | 100  78   75   85   90 | 93   96   100  97   93 | 11.4 12.0 61.9  26.7 362.5 |
|    | High   | 95   93  84(1)  87   95 | 22   26  100(1) 30   22 | 13.8 68.8 659.3 137.4(1) 76.1 |
|    | Extreme| 95   93  84(9)  87(4) 95 | 22   26  100(9) 30(4) 22 | 41.6 567.0 225.9(9) 708.1(4) 184.6 |
| 42 | Normal | 99   87   78   83   93 | 93   96   100  97   93 | 13.3 44.6 261.9  86.2 146.7 |
|    | High   | 99   89  80(5)  87   95 | 49   51  100(5) 52   49 | 16.9 38.7 1097.2(5) 240.6 36.9 |
|    | Extreme| 97   91  85(9)  88(7) 98 | 26   31  100(9) 50(7) 26 | 112.2 797.4 227.6(9) 1090.8(7) 165.1 |
5. Computational experiments

Results: representation of graph
# 5. Computational experiments

## Results

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\mu^{\text{lead}}$</th>
<th>$\mu^{\text{op}}$</th>
<th>$A[%]$</th>
<th>$B^t[\text{pcs.}]$</th>
<th>$B^p[%]$</th>
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<td>1.60</td>
<td>100.0</td>
<td>492.0</td>
<td>3.7%</td>
</tr>
</tbody>
</table>

$A[\%] = \text{Machine availability}$

\[
B^t = \sum_{\pi \in \Pi} \sum_{m \in M} (d_{m\pi} - z_{m\pi}^{\text{sim}})
\]

\[
B^p = \frac{B^t}{\sum_{\pi \in \Pi} \sum_{m \in M} d_{m\pi}}
\]
5. Conclusions

Overview

- Dynamic lot-sizing and operator-assignment problem for flexible, multi-stage production system
- Uncertainty considered: manual operation time, machine breakdown, etc
- Suggested formulation and simulation based approach

Further research

- Manufacturing lead time
- Related to CPS (cyber-physical production system)