Production control policy for tandem workstations with constant service times and queue time constraints


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Introduction

- Research motivation

  - Trade-off relationship between rework rate and capacity loss rate

  High WIP threshold $\rightarrow$ Low capacity loss rate
  $\rightarrow$ High queue time occurs $\rightarrow$ high rework rate

Optimal WIP threshold is needed for low capacity loss rate and low rework rate!
**Literature review**

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**Contribution**

- First research on analyzing the trade-off between capacity and rework rate.
Problem description

- **System configuration**
  - One machine at each stage
  - Two-stage flow shop with reworks under queue time limits
  - Considering machine breakdowns

Current practice

\[ \text{Safety WIP level (by jobs)} = \mu_2 \times t_c, \]
Problem description

- Detailed system configuration
  - Notation

\[
\begin{align*}
F_i & \quad \text{time to failure with mean } f_i, \ i = 1, 2 \ \text{where } i \ \text{refers to station } i; \\
R_i & \quad \text{repair time with mean } r_i, \ i = 1, 2 \ \text{where } i \ \text{refers to station } i; \\
v_i & \quad \text{availability of station } i \ (=f_i/(f_i + r_i)), \ i = 1, 2 \ \text{where } i \ \text{refers to station } i;
\end{align*}
\]

\[
\begin{align*}
t_c & \quad \text{length of queue time constraints (constant);} \\
s_i & \quad \text{constant service time of station } i, \ i = 1, 2;
\end{align*}
\]
Problem description

- Detailed system configuration
  - Notation

\[ l_2 \] number of queued jobs in front of Station 2 when Station 2 is down;

Safety WIP level (by jobs) = \( \mu_2 \times t_c \),

\[ = \frac{1}{S2}[\text{job/min}] \times 500 \text{ [min]} = \frac{1}{10} \times 500 = 50 \text{ jobs} \]

(Current practice)

\[ t_s \] safety WIP level by time (constant);

Ts should be equal or smaller than Tc in any rational setting to prevent rework. Ts \( \leq 500 \text{ min} \)
Problem description

- Assumptions

1. Interruptions are time-based preemptive

2. Station 1 stops sending jobs to station 2 when station 2 is down and restart sending jobs to station 2 once station 2 has recovered. Queued jobs at station 2 are always equal to the WIP level threshold when station 2 is down.

3. Station 1 stops processing new jobs right after the WIP level threshold is achieved

4. Station 1 and 2 will not have breakdowns at the same time

5. No batching or setup time

6. WIP is always sufficient at the first station
Assumptions

7. Single time window constraint

8. MTTF(mean time to failure) for each machine is greater than the mean service time

9. Dispatching rule; FIFO
Problem description

- **Problem description**
  - Decision variables
    - Optimal WIP level threshold (=safety WIP level)
  
- **Objective**
  - Minimizing total cost (=Minimizing rework rate, maximizing throughput)
    - Cost1; rework/scrap cost (material cost)
    - Cost2; capacity loss cost

- **Constraints**
  - Queue time constraint
Solution approach

- **Rework rate analysis**
  - **Conditions for different rework quantities**

\[
N_r = \begin{cases} 
0, & R_2 + l_2s_2 - t_c \leq 0, \\
\left\lfloor \frac{\Delta t_2}{s_2} \right\rfloor, & 0 < R_2 + l_2s_2 - t_c \leq l_2s_2, \\
l_2s_2 \leq R_2 + l_2s_2 - t_c, & 
\end{cases}
\]
Solution approach

- **Rework rate analysis**
  - **Conditions for different rework quantities**

![Diagram]

- \( T0 \)
- \( T1 \) (M2 수리기간)
- \( T2 \) (safety WIP level by time)
- \( T3 \)

\( L2 \cdot S2 = Ts \)

\( Tc \) (queue time limit)

①조건

②조건

③조건

\( N_r = \begin{cases} 
0, & \frac{R_2 + l_2s_2 - t_c}{l_2} \\
\frac{\Delta s}{s_2} & 0 < R_2 + l_2s_2 - t_c \leq l_2s_2, \\
\frac{R_2 + l_2s_2 - t_c}{l_2} & l_2s_2 \leq R_2 + l_2s_2 - t_c,
\end{cases} \)
Solution approach

- Capacity loss rate analysis
  - Conditions for different capacity loss rate

\[ \Delta t_l = \begin{cases} 
0, & 0 \leq R_1 < l_2s_2 - s_1, \\
(R_1 + s_1 - l_2s_2), & l_2s_2 - s_1 \leq R_1.
\end{cases} \]

Capacity loss rate or capacity loss duration
Solution approach

- Capacity loss rate analysis
  - Conditions for different capacity loss rate

```
: capacity loss duration = (R1 + S1) - (L2 * S2) [min]
```
Control policy

\[ TC(l_2) = \left( \frac{C_1}{N + m} \right) E[A_r] + \left( \frac{C_2}{N + k} \right) E[A_1] \]

\[ \frac{d(TC(l_2))}{dl_2} = \frac{s_2e^{\frac{l_2}{r_2}} \left( \frac{C_1}{N} + m \right)}{f_2 - s_1} - \frac{s_2e^{\frac{s_1-l_2}{r_2}} \left( \frac{C_2}{N} + k \right)}{v_2(f_1 + r_1)} \]

\[ \frac{d^2(TC(l_2))}{dl_2^2} = \frac{s_2e^{\frac{s_1-l_2}{r_2}} \left( \frac{C_2}{N} + k \right)}{r_1v_2(f_1 + r_1)} + \frac{s_2e^{\frac{l_2-s_1}{r_2}} \left( \frac{C_1}{N} + m \right)}{r_2(f_2 - s_1)} \]

\[ l_2 = \frac{\ln \left( \frac{C_2}{N + k} \right) \left( f_2 - s_1 \right)}{\ln \left( \left( \frac{C_1}{N + m} \right) v_2(f_1 + r_1) \right)} - \left( \frac{s_1}{r_1} + \frac{t_c}{r_2} \right) \]

Safety WIP level (by jobs) = \( \mu_2 \times t_c \),

Total cost 식

L2에 대해 정리

Proposed safety WIP 식

기존 safety WIP 식

L2에 대해 2번 편 미분 한 식이 0과 같다!
Computational experiment

- **Test result (1)**
  - Sensitivity analysis on service time ratio

  \[
  C_1 = 3,000,000, \quad C_2 = 20,000,000, \quad N = 96,000, \quad m = 1500 \text{ and } k = 3000. \\
  t_c = 12, \quad r_1 = 6, \quad r_2 = 8, \quad s_1 = 0.5, \quad f_1 = 120, \quad f_2 = 168, \quad s_2 \text{ is 0.6}
  \]

  Total cost가 가장 낮은 지점의 WIP threshold값=약15

  Total cost가 가장 낮은 지점의 WIP threshold값=약10

  두 station간의 성능 차가 많이 나는 경우

  S2=0.9로 변경한 경우
Computational experiment

- Test result (1) \( (S_2/S_1) \)
  - Sensitivity analysis on service time ratio

두 station간의 성능 차가 적게 나는 경우 (=service time ratio is small)

두 station간의 성능 차가 많이 나는 경우 (=service time ratio is large)

Not flat TC curve

WIP threshold를 아무렇게나 잡는다면, total cost가 급격히 올라갈 수 있다. (= service time ratio ↑ sensitivity ↑)
Computational experiment

- **Test result (2)**
  - Sensitivity analysis on unit cost

\[ m = \text{rework cost}, \ k = \text{capacity loss cost}, \ C_i = \text{annual capital cost of station } i \]

TC in the original settings.
\[ (s_2 = 0.9), \ m = 1500 \]

Unit cost value increased
\[ m = 5000, \]

Flat TC curve

( rework cost ↑ sensitivity ↑ )
Computational experiment

- Test result (3)
  - Sensitivity analysis on repair time

(sensitivity ↓)
Computational experiment

- **Test result (4)**
  - Simulation validation

\[ l_2 = \frac{\ln[(C_2/N + k)(f_2 - s_1)] - \ln[(C_1/N + m)v_2(f_1 + r_1)] + s_1/r_1 + t_c/r_2}{s_2(1/r_1 + 1/r_2)} \]

Proposed safety WIP 식

![Graph showing service time of station 2](image_url)
Conclusions

- **Summary**
  - System
    - 2-stage, 2-machine tandem workstations
    - Constant service times
    - Reworks under queue time limits
    - Random machine breakdowns.
  - Simulation based approach
    - Trade-off relationship between capacity loss rate & rework rate
  - Proposed new safety WIP model (control policy)

- **Further Research**
  - More complicated system
    - Multi product, multi queue time constraint
    - Setup. Batching
    - Multi station
  - Tradeoff between queue time and rework rate
Conclusions

- **Disadvantage**
  - Safety WIP approach for simplest time constrained model
    - Cannot use the model to real problems with various product, multiple queue time constraints

- **Advantage**
  - First research on analyzing the trade-off between capacity and rework rate.
Thank You!