A new silver-meal based heuristic for the single-item dynamic lot-sizing problem with returns and remanufacturing

Schulz, T.
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2. Problem Description
3. Adapted Silver-Meal Heuristic
4. Computational experiment
5. Improvement
6. Computational experiment
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1. Introduction

Dynamic lot singing model with returns and remanufacturing

(1) Single products
(2) No capacity restriction
(3) When and how to re/manufacture product for each period \( t \)
(4) 2 kinds of setup occur
2. Problem Description

- **Objective function**
  Minimize the total cost (setup costs and inventory holding costs)

- **Decision variables**
  1. When and how mulch to re/manufacture product for each period $t$
  2. Inventory level at the end of period

- **Assumptions**
  1. No initial inventory
  2. Backlogging is not allowed
  3. All parameters are to be nonnegative
  4. No lead time

- **Constraints**
  1. Inventory balance
  2. Setups

- **Approach**
  Formulation, Heuristics
2. Problem Description

Notation

Indices

t periods, \( t = 1, 2, \ldots, T \)

Parameters

\( K^R \) Setup cost for remanufacturing
\( K^M \) Setup cost for manufacturing
\( h^R \) unit inventory holding cost for recoverable inventory
\( h^M \) unit inventory holding cost for serviceable inventory
\( R_t \) number of return product
\( D_t \) demand for item type \( i \) for period \( t \)
\( Q \) sufficiently large number

Decision Variables

\( x^R_t \) amount of remanufacturing in period \( t \)
\( x^M_t \) amount of manufacturing in period \( t \)
\( y^R_t \) binary variables =1 if remanufacturing quantity is occured, 0 otherwise.
\( y^M_t \) binary variables =1 if manufacturing quantity is occured, 0 otherwise.
\( y^R_t \) inventory level for recoverable inventory at the end of period \( t \)
\( y^M_t \) inventory level for serviceable inventory at the end of period \( t \)
2. Problem Description

Formulation

Objective Function

\[
\min C = \sum_{t=1}^{T} \left( K^R \cdot y_t^R + K^M \cdot y_t^M + h^R \cdot y_t^R + h^M \cdot y_t^M \right)
\]

(1)

\[
y_t^R = y_{t-1}^R + R_t - x_t^R \quad \forall t = 1, \ldots, T
\]

(2)

\[
y_t^M = y_{t-1}^M + x_t^R + x_t^M - D_t \quad \forall t = 1, \ldots, T
\]

(3)

\[
x_t^R \leq Q \cdot y_t^R \quad \forall t = 1, \ldots, T
\]

(4)

\[
x_t^M \leq Q \cdot y_t^M \quad \forall t = 1, \ldots, T
\]

(5)

\[
y_0^R = y_0^M = 0
\]

(6)

\[
\gamma_t^R, \gamma_t^M \in \{0, 1\} \quad \forall i = 1, \ldots, T
\]

\[
y_t^R, y_t^M, x_t^R, x_t^M \geq 0 \quad \forall i = 1, \ldots, T
\]

Teunter et al. (2006) – NP-hard conjecture
3. Approach

Silver-Meal heuristic (1973)

Customers

Serviceable inventory

Manufacturing

\[ K_\tau + h \sum_{\tau}^{z} y_t \]

\[ \frac{z - \tau + 1}{C_{\tau,z}} \]
3. Approach

Adapted Silver-Meal heuristic

Option 1: Manufacture Only

\[ x^M_{\tau} = \sum_{i=\tau}^{z} D_i \]  \hspace{1cm} (7)

\[ C^1_{\tau,z} = \frac{K^M + h^M \cdot \sum_{l=\tau}^{z} y^M_{l} + h^R \cdot \sum_{l=\tau}^{z} y^R_{l}}{z - \tau + 1} \]  \hspace{1cm} (8)

Option 2: Remanufacture (and manufacture if necessary)

\[ x^R_{\tau} = \min \left( y^R_{\tau-1} + R_{\tau}, \sum_{l=\tau}^{z} D_l \right) \]
\[ x^M_{\tau} = \max \left( \sum_{l=\tau}^{z} D_l - y^R_{\tau-1} - R_{\tau}, 0 \right) \]  \hspace{1cm} (9)

\[ C^2_{\tau,z} = \frac{K^R + K^M \cdot y^M_{\tau} + h^M \cdot \sum_{l=\tau}^{z} y^M_{l} + h^R \cdot \sum_{l=\tau}^{z} y^R_{l}}{z - \tau + 1} \]  \hspace{1cm} (10)
3. Approach

**Adapted Silver-Meal heuristic**

Option 3: Manufacture first, remanufacture (in multiple lots) later

\[
NR_t = \sum_{i=\tau}^{t} (D_i - R_i) - y_{t-1}^R \quad \forall t = \tau + 1, \ldots, z.
\] (11)

\[
x_{\tau}^M = \max\left(D_{\tau}, \max_{t=\tau+1,\ldots,z} (NR_t)\right), \quad x_{\tau}^R = 0
\] (12)

\[
x_{t}^M = 0, \quad x_{t}^R = \max\left(\sum_{i=\tau}^{t} D_i - \sum_{i=\tau}^{t-1} x_{i}^R - x_{\tau}^M, 0\right) \quad \forall t = \tau + 1, \ldots, z
\] (13)

\[
C_{\tau,z}^3 = \frac{\sum_{t=\tau}^{z} y_{t}^R \cdot K^R + [K_{M}^M] + h_{M}^M \cdot \sum_{i=\tau}^{z} y_{i}^M + h^R \cdot \sum_{t=\tau}^{z} y_{t}^R}{z - \tau + 1}
\] (14)
3. Approach

 Improvement initial solution
(Decrease the number of setups)

Step 1: Find initial schedule
- Determine net requirements using Equation (11)
- Determine $x^M_i$ and $x^R_i$ using Equation (12)
- Determine $y^M_i$ and $y^R_i$ using Equation (13)
- Determine $y^M_i$ and $y^R_i$ using Equations (2) and (3)

$C_{ini} = C^3_{t,z}$

Step 2: Improve the initial schedule for periods $t$ to $z$
For $k = t + 1$ to $z$
If $x^R_i > 0$ then
$x^M_i = x^M_i + x^R_i, \quad x^R_i = 0$
For $i = t + 1$ to $k$
Update $x^R_i$ using Equation (13)
Update $y^M_i$ and $y^R_i$ using Equations (2) and (3)
Next $i$

Determine $\Delta C_t(k) = C^3_{t,z} - C_{ini}$
Reset initial schedule
Find period $l$ (period of the last remanufacturing lot before period $k$)
If $y^R_i \geq x^R_i$ then
$x^M_i = x^M_i + x^R_i, \quad x^R_i = 0$
Else
$x^M_i = x^M_i + (x^R_i - y^R_i), \quad x^R_i = y^R_i, \quad x^R_i = 0$
End If
Update $y^M_i$ and $y^R_i$ using Equations (2) and (3)
Determine $\Delta C_t(k) = C^3_{t,z} - C_{ini}$
End If
Next $k$

Step 3: Implement the best option
If $\min_{k \in [t+1, \ldots, z]} (\Delta C_t(k), \Delta C_t(k)) < 0$ then
Implement the best schedule which becomes the updated initial schedule
$C_{ini} = C_{ini} + \min_{k \in [t+1, \ldots, z]} (\Delta C_t(k), \Delta C_t(k))$, Goto Step 2:
End If
3. Approach

Adapted Silver-Meal heuristic

Option 4: Remanufacture first, manufacture (in multiple lots) later

\[ x_{\tau}^M = 0, \quad x_{\tau}^R = y_{\tau}^R + R_{\tau} \] (15)

\[ x_t^M = \max \left( \sum_{i=\tau}^{t} D_i - \sum_{i=\tau}^{t-1} x_i^M - x_{\tau}^R, 0 \right), \quad x_t^R = 0 \quad \forall t = \tau + 1, \ldots, z \] (16)

\[ C_{\tau,z}^4 = \frac{K_{R}^R + \sum_{t=\tau}^{z} y_t^M \cdot K_t^M + h_t^M \cdot \sum_{t=\tau}^{z} y_t^M + h_{R}^R \cdot \sum_{t=\tau}^{z} y_t^R}{z - \tau + 1} \] (17)
4. Computational experiments

Environment

- MIP solver: CPLEX 11.0

DATA

- $T = 12$
- $K^M / K^R = \{200, 500, 2000\}$
- $h^M = 1, h^R = 0.2, 0.5, 0.8$
- $D_t = DU(0, 100)$
- Return ratio = $\{30\%, 50\%, 70\%\}$

Performance measure

- %GAP

$$100 \left( \frac{Z^*(.) - v(.)}{Z^*(.)} \right)$$

$Z^*(.)$: optimal
$v(.)$: heuristic value

6480 instances for each combination
4. Computational experiments for initial solution

Results

Table 1. Performance of the $SM_2$ and $SM_4$ heuristic.

<table>
<thead>
<tr>
<th></th>
<th>Percentage cost error to the optimal solution</th>
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<tbody>
<tr>
<td></td>
<td>Average</td>
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<tr>
<td></td>
<td>$SM_2$</td>
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<tr>
<td>All instances</td>
<td>7.5%</td>
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<tr>
<td>Demand</td>
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<tr>
<td>Small variance</td>
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<tr>
<td>Large variance</td>
<td>7.8%</td>
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<tr>
<td>Returns</td>
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<td>Small variance</td>
<td>7.3%</td>
</tr>
<tr>
<td>Large variance</td>
<td>7.7%</td>
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<tr>
<td>Return ratio</td>
<td></td>
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<tr>
<td>30%</td>
<td>5.5%</td>
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<tr>
<td>50%</td>
<td>8.5%</td>
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<tr>
<td>70%</td>
<td>8.4%</td>
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<tr>
<td>$K^M$</td>
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<td>200</td>
<td>4.3%</td>
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<tr>
<td>500</td>
<td>5.4%</td>
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<td>2000</td>
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<tr>
<td>0.8</td>
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</tbody>
</table>
5. Computational experiments

**Improvement solution**

For $i = 1$ to $T$

If $x_{i}^{R} > 0$ then

Find period $n$ (period of the next manufacturing lot after period $i$)

If $i + 1 \leq n \leq T$ then

$x_{i}^{R} = x_{i}^{R} + \min(x_{n}^{M}, \min_{k \in \{i, \ldots, T\}}(y_{k}^{R}))$

$x_{n}^{M} = \max(x_{n}^{M} - \min_{k \in \{i, \ldots, T\}}(y_{k}^{R}), 0)$

Update $y_{i}^{M}$ and $y_{i}^{R}$ using Equations (2) and (3)

If total cost determined by Equation (1) cannot be reduced then

Reverse decisions made regarding $x_{i}^{R}$ and $x_{n}^{M}$

Update $y_{i}^{M}$ and $y_{i}^{R}$ using Equations (2) and (3)

End If

Else If $y_{i}^{M} > 0$ then

Find period $l$ (period of the last manufacturing lot before period $i$)

$x_{i}^{R} = x_{i}^{R} + \min(y_{i-1}^{M}, y_{i}^{M}, \min_{j \in \{i, \ldots, T\}}(y_{j}^{R}))$

$x_{i}^{M} = \max(y_{i-1}^{M} - \min_{j \in \{i, \ldots, T\}}(y_{j}^{R}), 0)$

Update $y_{i}^{M}$ and $y_{i}^{R}$ using Equations (2) and (3)

If total cost determined by Equation (1) cannot be reduced then

Reverse decisions made regarding $x_{i}^{R}$ and $x_{n}^{M}$

Update $y_{i}^{M}$ and $y_{i}^{R}$ using Equations (2) and (3)

End If

End If

End If

Next $i$
## 5. Computational experiments

### Results

Table 2. Performance of the $SM_2^+$ and $SM_4^+$ heuristic.

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>Standard deviation</th>
<th>Maximum</th>
<th>Average</th>
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5. Conclusions

Overview

- Dynamic lot-sizing problem for the single item
- Remanufacturing / manufacturing system with returned product
- Suggested formulation and heuristic

Further research

- More detailed modeling of the remanufacturing process
- Disposal option for the recoverable parts when they are not required
- To adapt rolling horizon environment
- Uncertainty demand