Weekly airline fleet assignment with homogeneity

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1. Introduction

• The airline schedule planning processes

• Fleet assignment problem (FAP) consists of determining the aircraft type to assign to each flight leg in order to maximize the total expected profits.

• This paper considers the weekly fleet assignment problem in the case where homogeneity of aircraft type is sought over legs sharing the same flight number.

• Improving customer service and the planning of operations
## 1. Introduction

- Literature review – addressing the classical fleet assignment problem over a daily horizon

<table>
<thead>
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<th>author</th>
<th>problem</th>
<th>approach</th>
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<td>Basic FAM</td>
<td>Abara, 1989</td>
<td>Applying integer linear programming to the fleet assignment</td>
<td>integer linear programming</td>
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<td>Subramanian et al, 1994</td>
<td>Coldstart: Fleet assignment at Delta airline</td>
<td>mixed integer programming</td>
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<td>Hane et al, 1995</td>
<td>The fleet assignment problem: solving a large-scale integer program</td>
<td>mixed linear integer programming,</td>
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<td>FAM with variable departure time</td>
<td>Desaulniers et al, 1997</td>
<td>Daily aircraft routing and scheduling</td>
<td>column generation</td>
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<td>Rexing et al, 2000</td>
<td>Airline fleet assignment with time windows</td>
<td>integer linear programming</td>
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</table>
2. Problem definition

- **Objective Function**
  
  Maximizing the total expected profits (for each leg)

- **Decision Variables**
  
  1. An aircraft of type k to assign to flight leg
  2. The number of aircraft of type k available on the ground at station
  3. The number of aircraft of type k used in the solution
  4. The dominant type for flight number
  5. The heterogeneous type for flight number

- **Constraints**
  
  1. Each leg is covered exactly once by a legal aircraft type
  2. Exactly one aircraft type is the dominant type for each flight number
  3. A penalty is imposed for each flight leg violating homogeneity
  4. Relations are the flow conservation constraints in each subnetwork

- **Assumption**
  
  1. The number of aircraft of each type is the same at the beginning and the end of the week.
  2. Maintenance scheduling cannot be taken into account at that time.
  3. A fixed cost is incurred to reflect the average weekly cost for own-ing an aircraft of type.
2. Problem definition

· Notation

\( K \) = the set of the different aircraft types
\( n^k \) = the number of available aircraft of type \( k \in K \)
\( S \) = the set of stations
\( S^k \) = the set of stations that can accommodate aircraft of type \( k \in K \)
\( L \) = the set of flight legs to cover = \( l \) or (o, d, t)

\( o = \text{origin} \)
\( d = \text{destination station} \)
\( t = \text{departure time} \)

\( f(l) \) = the flight number of leg \( l \in L \)
\( L^k \) = the set of legs to which an aircraft of type \( k \) can be assigned
\( K^l \) = the set of aircraft types that can be assigned to leg \( l \in L \)
\( F \) = the set of flight numbers
\( K^f \) = the set of aircraft types that can be assigned to a leg with flight number \( f \in F \)
\( \tau^k_{odt} \) = the arrival time of flight leg (o, d, t) \( \in L^k \)
\( \tau^k_{odt} = \text{flight departure time} + \text{flight duration} + \text{minimal connection time at the arrival station (depending on the aircraft type)} \)
2. Problem definition

· Parameter

\[ p_{lk}^k = \text{the profits expected from assigning an aircraft of type } k \in K \text{ to flight leg } l \in L^k \]

\[ c^k = \text{the average weekly fixed cost incurred for owning an aircraft of type } k \in K \]

\[ \gamma = \text{the penalty for each heterogeneous flight leg } l, \gamma > 0 \]

· Decision variable

\[ X_{lk}^k = \begin{cases} 1 & \text{if an aircraft of type } k \text{ is assigned to flight leg } l \\ 0 & \text{otherwise} \end{cases} \]

\[ Y_{ott}^k = \text{The number of aircraft of type } k \text{ available on the ground at station } o \text{ between the times } t \text{ and } t^+ \]

\[ E^k = \text{Each aircraft type } k \text{ to count the number of aircraft of type } k \text{ used in the solution} \]

\[ D_{kf}^k = \begin{cases} 1 & \text{if type } k \text{ is the dominant type for flight number } f \\ 0 & \text{otherwise} \end{cases} \]

\[ H_{lk}^k = \begin{cases} 1 & \text{if leg } l \text{ is heterogeneous (because aircraft of} \\
& \text{a non-dominant type has been assigned it)} \\ 0 & \text{otherwise} \end{cases} \]
2. Problem definition

· For Subnetwork

\[ G^k = (N^k, A^k) \]

\( N^k \) = node, potential event that may occur at station \( o \in S^k \) at time \( t \in T_o^k \), = (k, o, t)

\( A^k \) = containing two type of arcs: flight arcs (k, l) or (k, o, d, t)
  ground arcs (k, o, t, \( t^+ \))

\( A^k_F \) = The set of flight arcs associated with flights in operation at the chosen time

\( A^k_G \) = the set of ground arcs representing waiting at that time
3. Mathematical formulation

Maximize \[
\sum_{k \in K} \left( \sum_{l \in L^k} \left( p^k X^k_l - \gamma H^k_l \right) - c^k E^k \right)
\]  \hspace{1cm} (1)

subject to:

\[
\sum_{k \in K^f} X^k_l = 1 \quad \forall l \in L^k \quad \text{비행구간에 비행기종류 1개 배치}
\]  \hspace{1cm} (2)

\[
\sum_{k \in K^f} D^k_f = 1 \quad \forall f \in F^k \quad \text{k의 항공편에 대한 비행기종류}
\]  \hspace{1cm} (3)

\[
X^k_l - D^k_f(l) - H^k_l \leq 0 \quad \forall k \in K, l \in L^k
\]  \hspace{1cm} (4)

Flow conservation constraint

\[
\sum_{d \in S^k} \sum_{t' : o_{d_{k'}} = t} X^k_{d_{k'}} + Y^k_{o_{r-t}} - \sum_{d \in S^k} X^k_{o_{d_{k't}}} - Y^k_{o_{r-t}} = 0
\]  \hspace{1cm} (5)

The number of aircraft of type k

\[
\sum_{l \in A^k_l} X^k_l + \sum_{(k,o,t) \in A^k} Y^k_{o_{r-t}} = 0 \quad \forall k \in K
\]  \hspace{1cm} (6)

\[
0 \leq E^k \leq n^k \quad \forall k \in K
\]  \hspace{1cm} (7)

D : 1, H : 0 \rightarrow D : 0, H : 1

비행기종류 변경
3. Mathematical formulation

\[ \begin{align*}
Y_{\text{ott}+}^k & \geq 0 \quad \forall k \in K, \ (k, o, t) \in N^k \\
X_i^k & \in \{0, 1\} \quad \forall k \in K, \ l \in L^k \\
H_i^k & \in \{0, 1\} \quad \forall k \in K, \ l \in L^k \\
D_f^k & \in \{0, 1\} \quad \forall f \in F, \ k \in K^f
\end{align*} \]
4. Solution approaches

4.1 Direct MIP approach

The mixed-integer linear programming model (1)-(11) \[ \rightarrow \] CPLEX 6.5

- All binary variables (\( \geq 0.95 \)) are fixed to one
- Branching decision: D variables \( \rightarrow \) X variables
- Linear relaxation solution, a priority order is established for the decisions on the D variables
- Among the variables taking a fractional value, the closest to an integer value 0 or 1 has the highest priority
- The branch corresponding to fixing the selected variable to the nearest integer is explored first
- The decisions made on the X variables are managed by CPLEX
- Given large size of the search tree, a depth-first search strategy is used
- Given the high cost in terms of computational time, the first integer solution \( \rightarrow \) stop the exploration of the search tree
- At each node of the branch-and-bound search tree, the linear relaxation \( \rightarrow \) the dual simplex method
4. Solution approaches

4.2 Two-phase approach

1. A subset of the complete flight schedule → Dominant type → solution time ↓

※ A subset of the complete flight schedule = flights that appear in a selected horizon of n consecutive days of the week (n<7)

(1) a selection of n consecutive days of the week
(2) the model (1)-(11) restricted to the corresponding subset of flights MIP approach
(3) using penalized surplus and slack variables (due to feasibility of a periodic solution)
   → The model is modified
(4) its solution provides a dominant type for each flight numbers

F* = The set of these flight numbers
L* = The set of legs with a flight number in F*

2. Fixing the D variables of the flight numbers model (1)-(11) is solved
   (depending on the number of variables fixed, the second phrase can be solved rather rapidly)

(1) model (1)-(11) is reduced by fixing the D variables associated with the flight numbers in
(2) the values of variables are set according to the dominant types computed in the first phase
(3) putting the non-homogeneity penalty γ in the cost coefficient of the variables $X_i^k$
   allows to omit variables $H_i^k$

\[
\text{Maximize } \sum_{k \in K} \left( \sum_{l \in L} \bar{p}_i^k X_i^k - \sum_{l \in L \setminus L^*} \gamma H_i^k - c^k E^k \right) \quad (12)
\]

where
\[
\bar{p}_i^k = \begin{cases} 
  p_i^k - \gamma & \text{if } f(l) \in F^* \text{ and } D_{f(l)}^k \text{ is fixed at 0} \\
  p_i^k & \text{otherwise}
\end{cases}
\]
Maximize \[
\sum_{k \in K} \left( \sum_{l \in L^k} \tilde{p}_l^k X_l^k - \sum_{l \in L^k \setminus L^*} \gamma H_l^k - c^k E^k \right)
\]

where
\[
\tilde{p}_l^k = \begin{cases} 
    p_l^k - \gamma & \text{if } f(l) \in F^* \text{ and } D_{f(l)}^k \text{ is fixed at 0} \\
    p_l^k & \text{otherwise}
\end{cases}
\]

subject to:
\[
\sum_{k \in K^l} X_l^k = 1 \quad \forall l \in L
\]

\[
\sum_{d \in S^k} \sum_{t^*: d = t} X_{d^t}^k + Y_{o^r-t}^k - \sum_{d \in S^k} X_{o^d-t}^k - Y_{o^t+t}^k = 0
\]

\[
\forall k \in K, (k, o, t) \in N^k
\]

\[
\sum_{l \in A_l^k} X_l^k + \sum_{(k, o, t^*) \in A_{G}^k} Y_{o^t+t}^k - E^k = 0 \quad \forall k \in K
\]

\[
0 \leq E^k \leq n^k \quad \forall k \in K
\]

\[
Y_{o^t+t}^k \geq 0 \quad \forall k \in K, (k, o, t) \in N^k
\]

\[
X_l^k \in \{0, 1\} \quad \forall k \in K, l \in L^k
\]
4. Computational experiments

provided by Air Canada

Table 1
Data set characteristics

<table>
<thead>
<tr>
<th>Data set</th>
<th>Legs</th>
<th>Flight numbers</th>
<th>Legs per flight number</th>
<th>Aircraft types</th>
<th>Aircraft types per Leg</th>
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Table 2
Results for the direct MIP approach

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<th>Data set</th>
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<th>Number of heterogeneous legs</th>
<th>Optimality gap (%)</th>
<th>CPU time (s)</th>
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γ = Penalties for Heterogeneous flights
4. Computational experiments

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<th>CPU time phase 1 (s)</th>
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*a Averages for the seven possible choices of $n$ consecutive days.
4. Computational experiments

Fig. 1. Trade-off between homogeneity and profits (data set I).

Fig. 2. Trade-off between homogeneity and profits (data set II).
4. Computational experiments

<table>
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<th>( \gamma )</th>
<th>Number of heterogeneous legs</th>
<th>Relative profit gain (%)</th>
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<td>Air Canada</td>
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4. Conclusion

5. Advantage & Disadvantage

Thank you!