Ant colony optimization approach to a fuzzy goal programming model for a machine tool selection and operation allocation problem in an FMS

Felix T.S. Chan, Rahul Swarnkar (2006)
1. Introduction
2. Nature of the problem
3. Formulation
4. Ant systems
5. Solving the problem
6. Discussion
7. Conclusion
Introduction

FMS System description

• Goals
  – Productivity and products varieties

• Component
  – A number of multifunctional CNC machines
  – Limit tool capacity
  – Automated guided vehicles to take care material handling

• Processing characteristic
  – A part type includes one or more operations
  – Operations with specific tools on certain machines
  – Part types are to be processed by the system in batches (No grouping technology)
Proposed model

Problem description

- Problem
  - A machine-tool selection and operation allocation problem in an FMS
- Objective
  - A fuzzy goal to minimizing machining cost, setup cost, material handling cost.
- Decision variables
  - Selection of machines, assignment of the selected tools and the assignment of operation
- Assumption
  - The process details including tools, fixtures, etc. are available for each part type,
  - Each operation of a certain part type has to be assigned on one machine only.
  - Tool magazines of different machines may have different number of tool slots of identical shape and size.
  - As the slot requirements of a tool are independent of the part type, machines and tool magazine, it will occupy equal number of slots on each machine.
  - The tools cannot be transferred across the machines.
  - A tool cannot be duplicated in the same tool magazine.
- Constraints
  - Available time on machine
  - Tool life considering
  - Tool magazine capacity constraint
Formulation

Notations

- Parameters

\( P \) part type; \( 1 \leq p \leq P \), where \( P \) is the total number of part types

\( M \) machines; \( 1 \leq m \leq M \), where \( M \) is the number of machines

\( l \) tools; \( 1 \leq l \leq L \), where \( L \) is the number of tools

\( O \) operation for part type \( p \); \( 1 \leq o \leq O_p \), where \( O_p \) is the number of operation for part \( p \)

\( MS_{po} \) set of machines capable of performing operation \( o \) of part type \( p \)

\( TS_{po} \) set of tools for operation \( o \) of part \( p \)

\( T_{pmlo} \) machining time for operation \( o \) of part type \( p \) on machine \( m \) using tool \( l \)

\( C_{pmlo} \) machining cost for operation \( o \) of part type \( p \) on machine \( m \) using tool \( l \)

\( H_{mml} \) material handling cost for a part from machine \( m \)–\( m' \)

\( SU_m \) set up cost for machine \( m \)

\( T_m^a \) maximum allowable machine time on machine \( m \)

\( TL_l \) tool life of tool \( l \)

\( B_p \) batch size for part type \( p \)

\( TS_m \) maximum number of tools slots available on machine \( m \)

\( \mu_{po} \) machine selected for operation \( o \) of part type \( p \)

\( t_{po} \) tools selected for operation \( o \) of part type \( p \)

- Decision variables

\[ x_m = \begin{cases} 1 & \text{if machine } m \text{ is selected} \\ 0 & \text{otherwise} \end{cases} \]

\[ x_{m\mu_{po}} = \begin{cases} 1 & \text{if } m = \mu_{po} \\ 0 & \text{otherwise} \end{cases} \]

\[ z_{l_{po}} = \begin{cases} 1 & l = t_{po} \\ 0 & \text{otherwise} \end{cases} \]

\[ x_{pmlo} = \begin{cases} 1 & \text{if operation } o \text{ of part type } p \text{ is performed by tool } l \text{ of machine } m \\ 0 & \text{otherwise} \end{cases} \]

\[ y_{ml} = \begin{cases} 1 & \text{if tool } l \text{ is assigned to machine } m \\ 0 & \text{otherwise} \end{cases} \]
Integrated planning model

- Minimizing the following fuzzy goals:

1. Total machining cost $f_{MC} \leq G_{MC}$
   
   Where
   
   $$f_{mc} = \sum_{p=1}^{P} B_p \sum_{o=1}^{O_p} C_{pH_{po}H_{po+1}}.$$ 

2. Total set-up cost $f_{SU} \leq G_{SU}$
   
   where
   
   $$f_{SU} = \sum_{m=1}^{M} SU_{m}x_{m}.$$ 

3. Total set-up cost $f_{MH} \leq G_{MH}$
   
   where
   
   $$f_{MH} = \sum_{p=1}^{P} B_p \sum_{o=1}^{O_p-1} H_{pH_{po}H_{po+1}}.$$ 

Fuzzification of goals

$$\lambda_i = \begin{cases} 
1 & \text{if } f_i \leq G_i, \\
1 - \frac{f_i - G_i}{T_i} & \text{if } G_i \leq f_i \leq G_i + T_i, \\
0 & \text{if } f_i \leq G_i + T_i, 
\end{cases}$$

Where $f_i$ is the cost functions, $G_i$ the goals set by the decision maker, and $T_i$ is the tolerance limit for goals $i$.

Fig. Relationship between membership function and values of goals.
Formulation

Integrated planning model

- Subject to the following constraints:

\[
\sum_{m=1}^{M} y_{ml} = 1 \\
\sum y_{ml} \geq x_{m}
\]

\[
\sum_{p=1}^{P} \sum_{o=1}^{O_p} x_{pmlo} \geq y_{ml}(?) \quad \forall m, l
\]

\[
\sum_{p \in TS_{po}} \sum_{m \in MS_{po}} x_{pmlo} = 1 \quad \forall p, o
\]

\[
\sum_{p=1}^{P} B_p \sum_{o=1}^{O_p} T_{pmlo} z_{lt_{po}} \leq T_m^a \quad \forall m
\]

\[
\sum_{p=1}^{P} B_p \sum_{o=1}^{O_p} T_{pmlo} x_{mt_{po}} \leq TL_l \quad \forall l
\]

\[
\sum_{l=1}^{L} y_{ml} \leq TS_m \quad \forall m
\]

- A tool can be assigned to one machine, a machine can be assigned more than one tool
- Once a machine-tool combination is selected, the operations can be assigned to it
- Each operation should be processed
- Available processing time constraints
- Tool life constraints
- Tool magazine capacity
Solving the problem

- **ACO algorithm**

  - Concept of ant colony optimization algorithms

  **Flowchart**

  - **Start**
  - **Initialization**
  - Each ant completed tour
  - Update pheromone
  - Terminal Condition
  - **End**
Solving the problem

❖ ACO algorithm

• Node representation
  - \( n(p, o, m) \): part type \( o \): operation \( m \): machine

• Node selection
  - \( S_i \): set of node s to allowed to be visited
    \( S_i \) → Set of other operations' feasible nodes
  - \( I \) ants start at random node, and one time one unit movement.
  - Node selection is probabilistic:

  \[
  P_{jk}^i(t) = \begin{cases} 
  \frac{[\tau_{jk}(t)]^\alpha \eta_{jk}}{\sum l \in S_i [\tau_{lk}(t)]^\alpha \eta_{lk}} & \text{if } l \in S_i, \\
  0 & \text{otherwise,}
  \end{cases}
  \]

  \[
  \tau_{jk}(t + 1) = \rho \tau_{jk}(t) + \sum_{i=1}^{t'} \Delta \tau_{jk}^i
  \]

  \[
  \Delta \tau_{jk}^i = \begin{cases} 
  \frac{c}{MT_i} & \text{if } \text{ith ant uses edge}(j, k)\text{in its tours,} \\
  0 & \text{otherwise,}
  \end{cases}
  \]

• Tabu list
  The selected node \( n(p, o, m) \) is add to tabu list.
  → After a ant finish its visitation, tabu list specifies a feasible solution.

Notations/parameters in the algorithm

- \( N_i \): set of nodes to be visited
- \( S_i \): set of nodes allowed to be visited
- \( TL_i \): tabu list (set of nodes excluded from being visited)
- \( \tau_{jk}(t) \): intensity of pheromone trail on the edge \((j, k)\) at time \( t \)
- \( \alpha \): the relative importance of the trail; \( \alpha \geq 0 \)
- \( \beta \): the relative importance of the visibility; \( \beta \geq 0 \)
- \( \rho \): trail persistence, \( 0 \leq \rho \leq 1 \)
- \( Q \): a constant related to the quantity of trail laid by ants

where

- \( P_{jk} \): transition probability
- \( \eta_{jk} \): heuristic function obtained from a greedy heuristic
- \( \tau_{jk} \): intensity of the pheromone trail
- \( C \): positive constant
- \( MT_i \): machine time by the \( i \)th ant
## Solving the problem

### Numerical examples and computational experience

- **Data setting**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Range of values for generating random samples</th>
<th>Range of values for generating various goal sets</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Shop</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of machines in shop</td>
<td>5–7</td>
<td>No. of part types</td>
</tr>
<tr>
<td>Number of tools available on machine</td>
<td>140–280</td>
<td>Goal</td>
</tr>
<tr>
<td>Set-up cost for each machine</td>
<td>2–3</td>
<td>6–8</td>
</tr>
<tr>
<td>Material handling cost between machines</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>For ease of no movement of part</td>
<td></td>
<td></td>
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<tr>
<td><strong>Part type</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of part types in order</td>
<td>2–4</td>
<td>4–6</td>
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<tr>
<td>Number of operations in part type</td>
<td>25–40</td>
<td></td>
</tr>
<tr>
<td>Batch size of part type</td>
<td>3–9</td>
<td></td>
</tr>
<tr>
<td>Machining cost</td>
<td>4–9</td>
<td></td>
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<tr>
<td>Machining time</td>
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</table>

- **Table Content**

<table>
<thead>
<tr>
<th>Machining cost</th>
<th>Number of part types</th>
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<tr>
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<td>1250–1450</td>
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<td>6</td>
<td>1500–1700</td>
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<table>
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<td>7</td>
<td>950–1050</td>
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<td>8</td>
<td>1100–1300</td>
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Solving the problem

- Numerical examples and computational experience
  - A typical set of data

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<td>2.6</td>
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<td>2</td>
<td>3.6</td>
<td>5.5</td>
<td>2.3</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Material-handling cost between the machines

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<tbody>
<tr>
<td>1</td>
<td>2.3</td>
<td>2.1</td>
<td>2.4</td>
<td>2.1</td>
<td>2.3</td>
<td>0.1</td>
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<td>2.4</td>
<td>2.3</td>
<td>2.2</td>
<td>2.2</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.3</td>
<td>2.4</td>
<td>2.5</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2.3</td>
<td>2.1</td>
<td>0.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2.5</td>
<td>0.1</td>
<td></td>
<td></td>
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</tr>
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</table>

Goal set for various costs

<table>
<thead>
<tr>
<th>Type of cost</th>
<th>Goal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total machining cost</td>
<td>1100</td>
</tr>
<tr>
<td>Total set-up cost</td>
<td>900</td>
</tr>
<tr>
<td>Total material handling cost</td>
<td>400</td>
</tr>
</tbody>
</table>
Production and Logistics Information

Solving the problem

- Numerical examples and computational experience
  - Environment
    - C++ by Microsoft Visual Studio 6.0
    - IBM PC with Pentium 3 with 800 MHz 512MB RAM
  - 30 problems solved in ACO algorithm

<table>
<thead>
<tr>
<th>Part type</th>
<th>Machine m</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>3-2</td>
<td></td>
<td></td>
<td></td>
<td>5-1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>3-2</td>
<td>2-3</td>
<td></td>
<td>4-1</td>
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<td>3</td>
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<td></td>
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<td>4-1</td>
<td>6-3</td>
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</tr>
</tbody>
</table>

Element \( l-o \) at cell \((p,m)\) of table shows operation \( o \) of part type \( p \) is performed by machine \( m \) using tool \( l \).

Operational time and various costs associated with the solution

<table>
<thead>
<tr>
<th>Part type</th>
<th>Operation time for the part type</th>
<th>Cost of operations for the part types</th>
<th>Material handling costs for the part type</th>
<th>Total set-up cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>268</td>
<td>193.5</td>
<td>72.0</td>
<td>750</td>
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<tr>
<td>2</td>
<td>321</td>
<td>204.0</td>
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<tr>
<td>3</td>
<td>431</td>
<td>419.5</td>
<td>60.0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>328</td>
<td>282.0</td>
<td>211.5</td>
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</tbody>
</table>

Machine and tool utilization

<table>
<thead>
<tr>
<th>Machine</th>
<th>Utilization (in %)</th>
<th>Tool</th>
<th>Time in use</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>2</td>
<td>000</td>
</tr>
<tr>
<td>2</td>
<td>74.3</td>
<td>3</td>
<td>142</td>
</tr>
<tr>
<td>3</td>
<td>68.4</td>
<td>4</td>
<td>624</td>
</tr>
<tr>
<td>4</td>
<td>0.0</td>
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</tr>
<tr>
<td>5</td>
<td>84.8</td>
<td>6</td>
<td>134</td>
</tr>
<tr>
<td>6</td>
<td>63.0</td>
<td></td>
<td>000</td>
</tr>
</tbody>
</table>

Fig. 3. Best results obtained at different number of iterations.
ACO parameters decision

- Parameters $Q, \alpha, \beta, \rho$
  - $Q$ has negligible influence on the solution convergence time.
  - $\alpha \in \{0, 0.25, 0.33, 0.5, 1, 2, 3, 4\}$, $\beta \in \{0.5, 1, 2, 4, 8\}$
    - Select the best combination by the values of objective functions.
  - $\rho \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$
    - Too high $\rho$ results in situation called stagnation (all ants construct the same solution over and over again).
    - Select the $\rho$ rendered minimum computation time.
Conclusion

❖ Overview

• Problem
  – A machine-tool selection and operation allocation problem in an FMS
    ✓ Fuzzy goal programming-based approach
    ✓ Constraints pertaining tool life, tool magazine capacity and machine time

• suggest
  – Fuzzy IP model
  – Ant colony algorithms
    ✓ Discussion in parameter setting

❖ Adv. & Disadv.
THANK YOU