A tabu search algorithm for the heterogeneous fixed fleet vehicle routing problem

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(Computers&Operation Research, 2011)

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1. Introduction

- Vehicle Routing Problem (VRP)
  - Design of the optimal routes used by a fleet of vehicles, based at one or more depots, to serve a set of customers with known demand

- Vehicle
  - Homogeneous
  - Heterogeneous

- The Heterogeneous Fleet Vehicle Routing Problem (HVRP)
  - Fleet size
    - Limited (considering operation cost)
    - Unlimited (considering operation cost + fixed cost)

- In this paper
  - Considering heterogeneous fixed fleet vehicle routing to minimizing the sum of variable operation costs using tabu search algorithm
2. Problem Description

- **G=(V,E)**: a logistics network
  - **V**: The set of nodes \( V = \{0, 1, 2, \ldots, n\} \)
    - Nodes 1, 2, \ldots, \( n \) represent customers
    - Vertex 0 represents the depot
      - Departure and arrival base of each vehicle
    - \( q_i > 0 \) non-negative demand
  - **E**: the set of edges
    - \( E = \{(i, j) : i, j \in V, i \neq j\} \)
    - Each arc \((i, j) \in E\) a distance between customers \( i \) and \( j \)

- \( n_k \): the number of vehicles of each type
  - \( k \): each type of vehicle \( k (k=1,\ldots,T) \)
- \( Q_k \): the capacity of vehicles of each type
  - \( Q_1 < Q_2 \ldots < Q_T \)
- \( v_k \): the variable cost of vehicles of each type (directly proportional to the distance traveled)
  - \( v_1 < v_2 \ldots < v_T \)
- \( c_{ij} \): the traveling cost of each edge
  - \( c_{ij} = v_k d_{ij} \)
2. Problem Description

- **Objective function**
  - Minimize the sum of variable operation cost

- **Decision variable**
  - Assignment of vehicle travels

- **Constraint**
  - Customer demands must be satisfied
  - Vehicle’s route must not exceed its capacity
  - All vehicles start and end at the depot
  - Each customer is visited exactly once
  - Not exceed the capacity of the vehicle

- **Solution approach**
  - Tabu search algorithm

*Fig. 1. Heterogeneous vehicle routes: example.*
3. Tabu search algorithm

- Tabu heuristic structure

- Local search metaheuristic

- The method explores the solution space by moving at each iteration from a solution \( s \) to the best solution in a subset of its neighborhood \( N(s) \).

- Transition is done even though the best neighboring solution is worse than the given solution.

- Tabu list
  - To avoid cycling, solutions possessing some attributes of recently explored solutions are temporarily declared tabu or forbidden

- Aspiration level
  - A tabu move can be allowed if it creates a solution better than the best solution obtained so far
3. Tabu search algorithm

Application of TS

- Solution representation method
- Initial solution method
- Neighborhood generation method
- Definition of tabu moves with the tabu list size
- Termination condition
- Diversification and intensification
3. Tabu search algorithm

- Solution representation method
  - \( R_1, R_2, ..., R_k \) (\( R_k = (0, i_{v_1}, i_{v_2}, ..., 0) \)) : a set of routes
  - \( k \) : the number of vehicles
  - \( i_{v_l} \) : an index for the \( l \)th node on route \( v \)

- Initial solution method
  - A traveling salesman problem (TSP) tour including all the customer is constructed
  - The tour starts with the depot and each time a new customer is inserted, following their ordering number
  - The partition of the tour into feasible routes is performed as follows
    - The partition starts with the customer adjacent to the depot
    - The free vehicle with the smallest capacity equal to or greater than the customer’s demand is selected
    - In this route, the customers that are admissible in terms of capacity are included, one by one, following the sequence defined by the tour
    - In this last case, each of the unrouted customers in inserted in one of the existing routes where its insertion cost is lowest,
3. Tabu search algorithm

- Neighborhood generation method
  - Four types of neighborhood moves
    - Single insertion ($F_I$)
      - A candidate customer $x_i$ is removed from its current route (origin)
      - A trial insertion is made in any of the other routes (destination)
    - Double insertion ($F_D$)
      - The candidates are any pair of customers ($x_i, x_j$) belonging to the same route
      - The destination route should have at least one of the $\delta$-nearest neighbors of $x_i$ or $x_j$
    - Triple insertion ($F_T$)
      - A chain or segment of three consecutive customers ($x_i, x_j, x_k$) belonging to the same route
      - The destination route should have at least one of the $\delta$-nearest neighbors of $x_i$ or $x_j$ or $x_k$
      - Either in the current order of the chain or in the reverse order, i.e. $(x_k, x_j, x_i)$
    - swap ($F_S$)
      - Exchanging two customers belonging to two different routes
  
  Parameters $F_I$, $F_D$, $F_T$ and $F_S$ denotes the frequencies of the single, double and triple insertion and swap moves, respectively.

- Route improvement procedure
  - At the end of iteration, modified by one of the following procedures
    - New routes by the GENI
    - Current routes by US
    - 2-opt
3. Tabu search algorithm

- **Objective function**
  - Changing of the vehicle type driven by the objective function

  \[
  \text{Minimize } f(S') = \sum_{i=1}^{r}(c_i + P_l_i)
  \]
  
  - For any candidate solution \(S'\), the objective function is denoted by \(f(S')\)
  - \(r\) : the total number of routes in \(S'\)
  - \(c_i\) : the variable cost of route \(i\)
  - \(P\) : a penalty term
    > Initially \(P=1\), after iteration all solutions are infeasible \(P=2\), feasible then, \(P=P/2\)
  - \(l_i\) : the load excess in rout \(i\)

  Long term memory is used, the objective function, \(f_1(s')\), is giving by the following equation

  \[
  \begin{cases}
  f_1(S') = f(S') \iff f(S') \leq f(s) \\
  f_1(S') = f_1(S') + \beta \sqrt{\rho n} \rho / K \iff f(S') > f(s)
  \end{cases}
  \]
  
  - \(S\) : the current solution
  - \(\beta\) : a scaling parameter
  - \(D\) : the largest absolute difference in \(f_1(S)\) that has occurred between two consecutive iterations
  - \(\rho\) : the number of times that customer has been moved
  - \(K\) : the number of iterations executed so far
3. Tabu search algorithm

- Definition of tabu moves with the tabu list size
  - Two main purposes
    - To prevent the return to the most recent visited solutions in order to avoid cycle
    - To drive the search towards regions of the solution space not yet explored and with high potential of containing good solution
  - The initial $\theta$ is given by $n/2$
    - Whenever the middle of the cycle is reached, it is set as $\theta = \max(\theta/2, 7)$
    - At the end of the cycle it is defined as $\theta = \min(2\theta + 3, 0.6n)$

- Termination condition
  - A cycle ends
    - A given number of iterations ($K_{BL}$) without improving the best feasible or infeasible solution ($S_B$)
    - When the number of iterations without improving the best feasible solution reaches $3K_{BL}$
3. Tabu search algorithm

- **Detailed description of the TSA**
  - $K_L$: limit of the total number of iterations in a phase
  - $\beta$: a scaling parameter
  - $\theta$: the initial value of the tabu tenure
  - $\delta$: the neighborhood restriction
  - $P$: penalty term
  - $K_{BL}$: limit of $K_B$ in the current cycle
    - $K_B$: number of iterations since the beginning of without improving the best solution
  - $F_I, F_D, F_T, F_S$: the frequencies of the types of moves
  - $K_{BF}$: number of iterations since the best feasible solution was found

- **6 phases**
  - A phase is a set of cycles
    - Cycles number is not known in advance
    - A phase can be interrupted it the stopping criterion is reached
  - Set 1(fewer customers): six phases
  - Set 2(more customers): three phases
  - Phase 1 starts with the solution given by the giant tour
  - The other phases start with the best feasible solution found in the previous phase
    - Solving set 2, phase 1 was executed twice ($\beta = 0.013, 0.015$) and the best solution found was passed on to the next phase

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**Fig. 1. Flow chart of the TSA.**
3. Tabu search algorithm

- **Diversification**
  - Driving the search towards promising regions of solution space not yet explored
    - The use of several different phases
    - The change of the vehicle type driven by the objective function
    - Long term memory expressed in the modified objective function
    - Shaking
      - Towards the end of the search, when the best known solution is good, shaking it may be the only way of escaping from a local optimum and reaching better solution.
      - Applying (i) the solution should be already good, difficult to improve by other means
      - (ii) the algorithm should have enough iterations
      - At the beginning of phases 4 and 6 in set1, phases 2 and 3 with set2

  Step1. To start with one of the \( \lfloor r/2 \rfloor \) nearest customers of the depot

  Step2. These customers are removed from their present place and insert one by one in the beginning of each route
    - Starting with the highest route type and the nearest customer of the depot

  Step3. The next shaking is applied to the \( r \) routes of \( S \)
    - By using this different shaking, guarantee a higher diversification

- **Intensification**
  - A detailed exploration of some region of the solution space, usually in the vicinity of a good solution
    - The value of is reduced in the middle of the cycle in order to allow the intensification,
    - Route improvement procedure
4. Computational results

- The value used for the main parameters
  - The initial value of $\theta$ is the following: $n/2$ in phases 1, 3 and 5; $n/3$ in phases 2 and 6 and $n/2.5$ in phase 4.
  - The initial value of $\delta$ is the following: 1 (2 with set 2) in phases 1 and 5; 2 in phases 2 and 6; 5 in phase 3 and 10 in phase 4.
  - $K_{BL} = 15n$ if $n < 200$ and $K_{BL} = 30n$ if $n \geq 200$. $K_{BL}$ is increased by $2n$ at the end of the cycle.
  - $K_I = 3000\sqrt{n}$ in phases 1 and 6, and in the other phases $K_I = 2000\sqrt{n}$.
  - In the first four phases: $F_D = 10$ and $F_S = 5$, at the beginning of the cycle; $F_D = 5$ and $F_S = 10$, in the middle of the cycle; $F_T = \infty$. In phases 5 and 6: $F_S = 1$, at the beginning of the cycle; $F_S = 2$, in the middle of the cycle; $F_D = \infty$ and $F_T = 7$.
  - $\beta$ oscillates between 0.012 and 0.015, starting with $\beta = 0.012$ and increasing 0.001 at the end of each cycle. This is different in phase 1 because it is repeated and, in this case, the initial value of $\beta$ is the following: 0.013 in the first execution and 0.015 in the second one.

- The neighborhood restriction($\delta$)
  - In order to have influence in the search $\delta$ must be very small

- The maximum value, $\delta_L$
  - $\delta_L = \min(n - 1, 2C_r)$
  - In the middle of the cycle
    - $\delta = \min(\delta + 1, \delta_L)$
  - At the end of the cycle
    - $\delta = \min(\delta + 2, \delta_L)$
4. Computational results

- Benchmark problems from the literature

**Table 1**
Data for the problems of set 1.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Type of vehicle</th>
<th>Ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>Q_A</td>
<td>v_A</td>
</tr>
<tr>
<td>13</td>
<td>50</td>
<td>20</td>
</tr>
<tr>
<td>14</td>
<td>50</td>
<td>120</td>
</tr>
<tr>
<td>15</td>
<td>50</td>
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<td>75</td>
<td>20</td>
</tr>
<tr>
<td>19</td>
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<td>100</td>
</tr>
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<td>20</td>
<td>100</td>
<td>60</td>
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</table>

Ratio = 100 × (total demand/total capacity).

**Table 2**
Data for the problems of set 2.

<table>
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<th>Problem</th>
<th>Type of vehicle</th>
<th>Ratio (%)</th>
</tr>
</thead>
<tbody>
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<td>B</td>
</tr>
<tr>
<td></td>
<td>Q_A</td>
<td>v_A</td>
</tr>
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<td>50</td>
</tr>
<tr>
<td>H2</td>
<td>240</td>
<td>50</td>
</tr>
<tr>
<td>H3</td>
<td>280</td>
<td>50</td>
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<tr>
<td>H4</td>
<td>320</td>
<td>50</td>
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<tr>
<td>H5</td>
<td>360</td>
<td>50</td>
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</table>

Ratio = 100 × (total demand/total capacity).
4. Computational results

- Result on instances from the literature

<table>
<thead>
<tr>
<th>Problem</th>
<th>n</th>
<th>Best known</th>
<th>Our best</th>
<th>Taillard</th>
<th>Tarantilis et al.</th>
<th>Li et al.</th>
<th>TSA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Cost</td>
<td>Cost</td>
<td>Cost</td>
<td>Cost</td>
<td>Cost</td>
<td>Cost</td>
</tr>
<tr>
<td>13</td>
<td>50</td>
<td>1517.84</td>
<td>1517.84</td>
<td>1536.55</td>
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<td>1519.96</td>
<td>843</td>
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<td>50</td>
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<td>368</td>
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<td>16</td>
<td>50</td>
<td>1144.94$^a$</td>
<td>1144.94</td>
<td>1159.14</td>
<td>350</td>
<td>1145.52</td>
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<tr>
<td>17</td>
<td>75</td>
<td>1061.96$^a$</td>
<td>1061.96</td>
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<tr>
<td>18</td>
<td>75</td>
<td>1823.58$^a$</td>
<td>1823.58</td>
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<td>971</td>
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<td>19</td>
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<td>1117.51$^b$</td>
<td>1120.33</td>
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<td>20</td>
<td>100</td>
<td>1534.17$^a$</td>
<td>1534.17</td>
<td>1592.16</td>
<td>3402</td>
<td>1556.35</td>
<td>1156</td>
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<tr>
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<td></td>
<td>1227.85</td>
<td>1228.21</td>
<td>1259.95</td>
<td>2011</td>
<td>1236.21</td>
<td>607</td>
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<td>Av. deviation (%)</td>
<td>0.93</td>
<td>2.61</td>
<td>-</td>
<td>0.68</td>
<td>-</td>
<td>0.03</td>
<td>-</td>
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<tr>
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<td>7</td>
<td>0</td>
<td>-</td>
<td>1</td>
<td>-</td>
<td>7</td>
<td>-</td>
</tr>
</tbody>
</table>

Values in boldface—identical to the best known.

- $^a$ Solution cost taken from Li et al. [6].
- $^b$ Solution cost taken from Taillard [1].

<table>
<thead>
<tr>
<th>Problem</th>
<th>n</th>
<th>Best known</th>
<th>Our best</th>
<th>Li et al.</th>
<th>TSA</th>
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</thead>
<tbody>
<tr>
<td></td>
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<td>Cost</td>
<td>Cost</td>
<td>Cost</td>
<td>Cost</td>
</tr>
<tr>
<td>H1</td>
<td>200</td>
<td>12067.65</td>
<td>12050.08</td>
<td>12067.65</td>
<td>12050.08</td>
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<tr>
<td>H2</td>
<td>240</td>
<td>10234.40</td>
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<td>280</td>
<td>16231.80</td>
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<td>16230.21</td>
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<td>17576.10</td>
<td>17458.65</td>
<td>17576.10</td>
<td>17458.65</td>
</tr>
<tr>
<td>H5</td>
<td>360</td>
<td>21850.40</td>
<td>21766.66 (21757.26)</td>
<td>21850.40</td>
<td>23220.72 (21852.36)</td>
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<tr>
<td>Average</td>
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<td>14027.49</td>
<td>1344</td>
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<td>0.00</td>
<td>-</td>
<td>-0.26</td>
<td>-</td>
</tr>
<tr>
<td>NB</td>
<td>4</td>
<td>0</td>
<td>-</td>
<td>4</td>
<td>-</td>
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Values in boldface—identical to the best known or better (in this last case they are also in italic).

- $^d$ This average is only for the first four problems.
4. Computational results

- New test problem
  - Established in a way similar to that used by Taillard

<table>
<thead>
<tr>
<th>Problem</th>
<th>( \pi )</th>
<th>( n )</th>
<th>( \pi_A )</th>
<th>( \pi_B )</th>
<th>( \pi_C )</th>
<th>( \pi_D )</th>
<th>( \pi_E )</th>
<th>( \pi_F )</th>
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<td>N1</td>
<td>150</td>
<td>50</td>
<td>1</td>
<td>5</td>
<td>100</td>
<td>1.5</td>
<td>4</td>
<td>150</td>
</tr>
<tr>
<td>N2</td>
<td>199</td>
<td>50</td>
<td>1</td>
<td>8</td>
<td>100</td>
<td>1.5</td>
<td>6</td>
<td>150</td>
</tr>
<tr>
<td>N3</td>
<td>120</td>
<td>50</td>
<td>1</td>
<td>6</td>
<td>100</td>
<td>1.5</td>
<td>3</td>
<td>150</td>
</tr>
<tr>
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<td>50</td>
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<td>4</td>
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<td>1.6</td>
<td>4</td>
<td>180</td>
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<td>900</td>
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<td>5</td>
<td>1500</td>
<td>1.5</td>
<td>3</td>
<td>2000</td>
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Ratio = 100 \times \frac{\text{total demand}}{\text{total capacity}}.

<table>
<thead>
<tr>
<th>Problem</th>
<th>( \pi )</th>
<th>FSMVRP solution</th>
<th>HFFVRP solution</th>
<th>Gap (%)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Cost</td>
<td>Vehicle fleet</td>
<td>Cost</td>
</tr>
<tr>
<td>N1</td>
<td>150</td>
<td>2220.01</td>
<td>5A, 2B, 3C, 7D</td>
<td>2243.76</td>
</tr>
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<td>199</td>
<td>2827.76</td>
<td>2A, 8, 4C, 12D</td>
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<td>134</td>
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<td>7A, 8, 4C, 2D</td>
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<tr>
<td>Average</td>
<td></td>
<td></td>
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</tbody>
</table>
5. Conclusion

- Conclusion
  - The TSA was the first tabu search algorithm applied to the heterogeneous fixed fleet vehicle routing problem
  - The results have proven that this metaheuristic is appropriate
    - To find high quality solution in a reasonable computing time

- Contribution
  - Playing the δ–neighborhood
  - The use of long term memory
  - The use of shaking

- 장점

- 단점
Thank You!

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