A dynamic inventory model with supplier selection in a serial supply chain structure

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3. Analysis of transportation freight rates
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1. Introduction

Background

- Dynamic lot-sizing model with supplier selection in a serial supply chain structure

1. Lead-Time is exist

2. Each stage has capacity constraints.

Fig. 1. A dynamic serial supply chain network with multiple suppliers and customers (network $G_D$).
1. Introduction

Literature review


Michael Florian and Morton Klein (1971) *Management Science* - Inventory decomposition property & capacity


Supplier selection, Lead Time
Capacity restriction for all stages
2. Problem Description

- **Objective Function:**
  Minimizing of the total cost (purchasing cost, shipping costs (fixed / variable), production cost, holding costs)

- **Decision Variables:**
  - replenishment order quantity (in a raw material)
  - production quantity
  - shipping quantity of replenishment order (in units of finished product)
  - inventory level

- **Assumption:**
  1. Backlogging is not allowed
  2. Product structure (BOM) = 1

- **Approach:**
  Formulation developed
2. Problem Description

\[ J = \{1,2,\ldots,n_J\} \text{ : set of supplier} \]
\[ K = \{1,2,\ldots,n_K\} \text{ : set of supply chain} \]
\[ S = \{1,2,\ldots,n_S\} \text{ : set of customer} \]
\[ T = \{1,2,\ldots,n_T\} \text{ : set of period} \]

\[ G_D = (N_D,A_D) \]
\[ N_D = J \cup K \cup S \]
\[ A_D = \{(j, 1) : j \in J\} \cup \{(k, k + 1) : k \in K/\{n_K\}\} \cup \{(n_K, s) : s \in S\} \]

Fig. 1. A dynamic serial supply chain network with multiple suppliers and customers (network $G_D$).
\[ J = \{1, 2, \ldots, n_J\} : \text{set of supplier} \]
\[ K = \{1, 2, \ldots, n_K\} : \text{set of supply chain} \]
\[ S = \{1, 2, \ldots, n_S\} : \text{set of customer} \]
\[ T = \{1, 2, \ldots, n_T\} : \text{set of period} \]
Problem Description

2. Problem Description

$J = \{1,2,\ldots,n_J\}$ : set of supplier  
$K = \{1,2,\ldots,n_K\}$ : set of supply chain  
$S = \{1,2,\ldots,n_S\}$ : set of customer  
$T = \{1,2,\ldots,n_T\}$ : set of period

$G_S = (N_S,A_S)$

$N_S = \{(0,j,t) : j \in J, t \in T\} \cup \{(k,t) : k \in K, t \in T\}$

$A_S = \{((0,j,t),(1,t+l_{0j})) : j \in J, t \in \{1,2,\ldots,n_T-l_{0j}\}\}$

$\cup\{((k,t),(k+1,t+l_k)) : k \in K \setminus \{n_K\}, t \in \{1,2,\ldots,n_T-l_k\}\}$

Elimination of unreachable arcs and nodes

$G'_S = (N'_S,A'_S)$

$N'_S \subseteq N_S$  
$A'_S \subseteq A_S$
Problem Description

Content overview

[MLSP-PC]

{Original [P]}

Non-linear transportation cost function by LTL

{Multi-level}

Approximation transportation cost

Piecewise linear transportation cost
Problem Description – characteristics

Theorem 1. For a given stage $k \in K$, let $m_k$ be either zero or the closest preceding stage with positive initial inventory or pending order, i.e.,

$$m_k = \max\{0, k' : y_{k'-1}^0 + i_k^0 > 0, k' \in \{1, \ldots, k\}\}.$$

Let $m_k^*$ be either $n_k$ or the closest succeeding stage from stage $k$ with positive ending inventory, i.e.,

$$m_k^* = \min\{n_k, k' : i_{k'}^n > 0, k' \in \{k, \ldots, n_k - 1\}\}.$$

In addition, $l_0 = \min\{l_{0j} : j \in J\}$.

Finally, let $y_0^0 = \sum_{j \in J} q_{0j}^0$.

Then, node$(k,t)$ is a feasible node in the (time-expanded)static network if and only if $t \in T_k$,

$$T_k = \left\{ t : 1 + \sum_{k'=m_k}^{k-1} l_{k'} \leq t \leq n_T - \sum_{k'=k}^{m_k^*-1} l_{k'}, t \in T \right\}.$$
2. Problem Description – characteristics

**Corollary 1.1.** A raw material order submitted by the manufacturing stage (stage 1) to a given supplier \( j \in J \) at time \( t \) will be feasible if and only if \( t \in T_{0j} \), where

\[
T_{0j} = \{ t : 1 \leq t \leq \max\{ t' \in T_1 \} - l_{0j} \}.
\]

**Lemma 1.** For a given stage \( k \in K \), if node \((k, 1)\) is connected to node \((n_K, t)\) at the demand stage \( n_K \) in period \( t \in T_k \), then every node \((k, 1)\), such that \( k \leq \hat{k} \leq K \), is also connected to node \((n_K, t)\).

**Theorem 2.** The supply chain inventory problem with supplier selection can only be feasible if for every demand node \((n_K, t)\),

\[
1 + \sum_{k = m_{n_K}}^{n_K - 1} l_k \leq t < 1 + \sum_{k' = 0}^{n_K - 1} l_{k'},
\]

the following condition holds:

\[
\sum_{k' = k_t}^{n_K} \left( y_{k' - 1}^0 + t_{k'}^0 \right) - \sum_{t' = 1}^{t - 1} d_{t'} \geq d^t,
\]

where, \( k_t = \min\{ n_K, k \in K : \sum_{k' = k}^{n_K - 1} l_{k'} < t \} \).
2. Problem Description – *characteristics*

\[ G'_s = (N'_s, A'_s) \]

\[ N'_s \subseteq N_s \quad N'_s = \{(0, j, t) : j \in J, t \in T_{0j}\} \cup \{(k, t) : k \in K, t \in T_k\}, \]

\[ A'_s \subseteq A_s \quad A'_s = \{((0, j, t), (1, t + l_{0j})) : j \in J, t \in T_{0j}\} \]
\[ \quad \{((k, t), (k + 1, t + l_k)) : k \in K \setminus \{n_K\}, t \in T_k\}. \]
2. Problem Description

Formulation [MLSP-PC]

Notation

Remaining Parameters

- $h^t_k$: unit holding cost at stage $k$ from period $t$ to period $t+1$, $k \in K$, $t \in T_k$.
- $b^t_{0j}$: capacity (in units) of supplier $j$ at time $t$, $j \in J$, $t \in T_{0j}$.
- $b^t_1$: production capacity (in units) at stage 1 in period $t$, $t \in T_1$.
- $b^t_k$: distribution capacity at stage $k$ in period $t$ (in units), $k \in K$, $t \in T_k$.
- $r^t_k$: inventory capacity (in units) at stage $k$, $k \in K$.
- $p^t_{0j}$: unit price of raw material for supplier $j$ in period $t$, $j \in J$, $t \in T_{0j}$.
- $p^t_1$: unit production cost in period $t$, $t \in T_1$.
- $f^t_{0j}$: setup cost for an order submitted to supplier $j$ in period $t$, $j \in J$, $t \in T_{0j}$.
- $f^t_1$: setup cost for production at stage 1 in period $t$, $t \in T_1$.
- $f^t_k$: setup cost at stage $k$ in period $t$, $k \in K_D$, $t \in T_k$.
- $a_j$: perfect rate of supplier $j$ (probability that a unit is acceptable), $j \in J$.
- $a$: minimum acceptable perfect rate.
2. Problem Description

Formulation [MLSP-PC]

Notation (Continued)

Decision variables

\[ q_{0j}^t \] replenishment order quantity (in raw material units) shipped from supplier \( j \) to stage 1 in period \( t \), \( j \in J, t \in T_{0j} \).

\[ x_1^t \] production lot size (in units of finished product) at the manufacturing stage (stage 1) at the beginning of period \( t \), \( t \in T_1 \).

\[ y_k^t \] replenishment order quantity (in units of finished product) shipped from stage \( k \) to stage \( k + 1 \) in period \( t \), \( k \in K_D, t \in T_k \).

\[ i_k^t \] inventory level (in units) held at stage \( k \) from period \( t \) to period \( t + 1 \), \( k \in K, t \in T_k \).

\[ w_{0j}^t \] 1 if supplier \( j \) receives a replenishment order at time \( t \); 0 otherwise; \( j \in J, t \in T_{0j} \).

\[ w_1^t \] 1 if a production order is submitted at time \( t \); 0 otherwise; \( t \in T_1 \).

\[ w_k^t \] 1 if a replenishment order is shipped from stage \( k \) to stage \( k + 1 \) at time \( t \); 0 otherwise; \( k \in K_D, t \in T_k \).
Problem Description

Notation (Continued)

Cost function components

- \(s_{0j}^t (q_{0j}^t)\) purchasing cost of \(q_{0j}^t\) units purchased from supplier \(j\) in period \(t, j \in J, t \in T_{0j}\).
  \[f_{0j}^t w_{0j}^t + p_{0j}^t q_{0j}^t,\]

- \(c_1^t (x_1^t)\) production cost of \(x_1^t\) units at stage 1 in period \(t \in T_1\).
  \[f_1^t w_1^t + p_1^t x_1^t,\]

- \(h_k^t (i_k^t)\) inventory holding cost of \(i_k^t\) units stored at stage \(k\) from period \(t\) to period \(t + 1, k \in K, t \in T_k\).
  \[h_k^t i_k^t,\]

- \(u_{1j}^t (q_{0j}^t)\) holding cost for in-transit inventory of \(q_{0j}^t\) units shipped from supplier \(j\) (stage 0) to stage 1 for periods \(t\) to \(t + l_{0j} - 1, j \in J, t \in T_{0j}\). It is calculated as
  \[\sum_{t' = t}^{t + l_{0j} - 1} h_1^t (q_{0j}^t).\]

- \(u_{k+1}^t (y_k^t)\) holding cost for in-transit inventory of \(y_k^t\) units shipped from stage \(k\) to stage \(k + 1\) for periods \(t\) to \(t + l_k - 1, k \in K_D, t \in T_k\). It is calculated as
  \[\sum_{t' = t}^{t + l_k - 1} h_{k+1}^t (y_k^t).\]

- \(g_{0j}^t (q_{0j}^t)\) transportation cost of \(q_{0j}^t\) units of row material shipped from supplier \(j\) to stage 1 in period \(t, j \in J, t \in T_{0j}\).

- \(g_k^t (y_k^t)\) transportation cost of \(y_k^t\) units of product shipped from stage \(k\) to stage \(k + 1\) in period \(t, k \in K_D, t \in T_k\).
2. Problem Description

Formulation \([\text{mixed integer nonlinear programming model (MINLP)}]\)

\[
\min Z = \sum_{j \in J} \sum_{t \in T_{0j}} s_{0j}^t (q_{0j}^t) + \sum_{t \in T_1} c_1^t (x_1^t) + \sum_{k \in K} \sum_{t \in T_k} h_k^t (i_k^t) + \sum_{j \in J} \sum_{t \in T_{0j}} u_{ij}^t (q_{0j}^t) + \sum_{k \in K_D} \sum_{t \in T_k} u_{k+1}^t (y_k^t)
\]

- Purchasing cost
- Production cost
- Holding cost
- Holding cost for in transit
- Holding cost for in transit
- Transportation cost for in transit
- Transportation cost for in transit
- Inventory balance between raw material and finished goods at manufacturing level
- Inventory balance for finished goods between consecutive stages
- Acceptable quality

Subject to

\[
\sum_{j \in J_t} q_{0j}^{t-1} + i_1^{t-1} = x_1^t + i_1^t, \quad t \in T_1,
\]

\[
x_1^t + i_2^{t-1} = y_2^t + i_2^t, \quad t \in T_2,
\]

\[
y_{k-1}^{t-1} + i_k^{t-1} = y_k^t + i_k^t, \quad k \in K_D \setminus \{2\}, t \in T_k,
\]

\[
y_{n_k-1}^{t-1} + i_{n_k}^{t-1} = d^t + i_{n_k}^t, \quad t \in T_{n_k},
\]

\[
\sum_{j \in J_t} a_j q_{0j}^{t-1} \geq a \sum_{j \in J_t} q_{0j}^{t-1}, \quad t \in T_1,
\]
2. Problem Description

Formulation [mixed integer nonlinear programming model (MINLP)]

Subject to (continued)

\[ q_{0j}^t \leq b_{0j}^t w_{0j}^t, \quad j \in J, t \in T_{0j}, \quad (7) \]
\[ x_1^t \leq b_1^t w_1^t, \quad t \in T_1, \quad (8) \]
\[ y_k^t \leq b_k^t w_k^t, \quad k \in K_D, t \in T_k, \quad (9) \]
\[ i_k^t \leq r_k, \quad k \in K, t \in T_k, \quad (10) \]
\[ i_k^t \geq 0, \quad k \in K, t \in T_k, \quad (11) \]
\[ q_{0j}^t \geq 0, w_{0j}^t \in \{0, 1\}, \quad j \in J, t \in T_{0j}, \quad (12) \]
\[ x_1^t \geq 0, w_1^t \in \{0, 1\}, \quad t \in T_1, \quad (13) \]
\[ y_k^t \geq 0, w_k^t \in \{0, 1\}, \quad k \in K_D, t \in T_k. \quad (14) \]
3. Analysis of transportation freight rates

Analysis of transportation freight rates

Fig. 3. Nominal freight transportation cost function.

Fig. 4. Actual freight transportation cost function.
Approximations to the transportation cost function

4. Approximations to the transportation cost function

Power approximation

Tyworth and Ruiz-Torres (2000)

\[ F_p(q) = a(q)^b, \]  

(22)

where \( a > 0 \) and \( -1 < b < 0 \) correspond to the coefficients of the nonlinear regression function.

Ventura and Mendoza (2008)

\[ \ln(F_p(q)) = \ln(a) + b \ln(q). \]  

(23)

Quadratic approximation

\[ F_q(q) = a + bq + cq^2, \]  

(24)

where \( a > 0 \), \( b > 0 \), and \( c \geq 0 \) correspond to the coefficients of the linear regression function.
Approximations to the transportation cost function

4. Approximations to the transportation cost function

**Fig. 5.** Graphical comparison of power and quadratic approximation functions with the actual freight cost function.
5. Alternative mixed integer linear programming model

- The set of freight ranges of the transportation cost function from supplier $j$ to stage 1:
  \[ R_{0j} = \{1, 2, \ldots, n_{R_{0j}}\} \]

- The set of freight ranges of the transportation cost function from stages $k$ and $k+1$:
  \[ R_k = \{1, 2, \ldots, n_{R_k}\} \]

- A flow variable, $q_r^r$, which assumes the value of the respective replenishment order quantity $q$, when it falls in transportation range $r$, and is 0 otherwise.

- The respective lower and upper bounds for range $r$:
  \[ \beta^{r-1} \text{ and } \beta^r \]

- A binary variable denoting whether ($\phi^r=1$) or not ($\phi^r=0$)

**Fig. 6.** Piecewise linear transportation cost function.
5. Alternative mixed integer linear programming model

Additional parameters

\( v_{0j}^r \) Fixed charge for the actual freight rate for supplier \( j \) when order quantity falls in range \( r, j \in J, r \in R_{0j}^f \).

\( v_k^r \) Fixed charge for the actual freight rate from stage \( k \) to stage \( k + 1 \) if order quantity falls in range \( r, k \in K_D, r \in R_k^f \).

\( e_{0j}^r \) Constant charge per unit for the actual freight rate for supplier \( j \) when order quantity falls in range \( r, j \in J, r \in R_{0j}^c \).

\( e_k^r \) Constant charge per unit for the actual freight rate from stage \( k \) to stage \( k + 1 \) if order quantity falls in range \( r, k \in K_D, r \in R_k^c \).
5. Alternative mixed integer linear programming model

Additional variables

$q_{0j}^{t,r}$ 1 if the replenishment order quantity (in units of raw material) sent from supplier $j$ to stage 1 in period $t$ falls in transportation range $r$; 0 otherwise; $j \in J, t \in T_{0j}, r \in R_{0j}$.

$q_{k}^{t,r}$ 1 if the order quantity (in units) sent from stage $k$ to stage $k+1$ in period $t$ falls in transportation range $r$; 0 otherwise; $k \in K_D \cup \{n_K\}, t \in T_k, r \in R_k$.

$q_{0j}^{t,r}$ defined as $q_{ij}^{t}$ when replenishment order quantity (in units of raw material) shipped from supplier $j$ in period $t$ falls in transportation range $r$; 0 otherwise; $j \in J, t \in T_{0j}, r \in R_{0j}$. Notice that $q_{0j}^{t} = \sum_{r \in R_{0j}} q_{0j}^{t,r}$.

$y_{k}^{t,r}$ defined as $y_{k}^{t}$ when replenishment order quantity (in units of finished product) shipped from stage $k$ to stage $k+1$ in period $t$ falls in transportation range $r$; 0 otherwise; $k \in K_D, t \in T_k, r \in R_k$. Notice that $y_{k}^{t} = \sum_{r \in R_k} y_{k}^{t,r}$. 
5. Alternative mixed integer linear programming model

Formulation [mixed integer nonlinear programming model (MILP)]

\[
\min \quad Z = \sum_{j \in J} \sum_{t \in T_{0j}} \left( f_{0j} w_{0j}^t + p_{0j} \sum_{r \in R} q_{0jr}^t \right) + \sum_{t \in T_2} \left( f_1 w_1^t + p_1 x_1^t \right) + \sum_{k \in K} \sum_{t \in T_k} h_k^t t_k + \sum_{j \in J} \sum_{t \in T_{0j}} \sum_{r \in R_{0j}} u_{ij}^t \sum_{r \in R_{0j}} q_{0jr}^t
\]

\[
+ \sum_{k \in K_D} \sum_{t \in T_k} \sum_{r \in R_k} y_{k+1}^t \sum_{r \in R_k} r_{k}^t \sum_{r \in R_k} r_{k}^t
\]

\[
+ \sum_{j \in J} \sum_{t \in T_{0j}} \sum_{r \in R_{0j}} v_{0jr}^t \rho_{0jr}^t + \sum_{j \in J} \sum_{t \in T_{0j}} \sum_{r \in R_{0j}} e_{0jr}^t q_{0jr}^t + \sum_{k \in K_D} \sum_{t \in T_k} \sum_{r \in R_k} v_{k}^t \rho_{k}^t
\]

\[
+ \sum_{k \in K_D} \sum_{t \in T_k} \sum_{r \in R_k} e_{k}^t y_k^t
\]

\[
 \text{Subject to}
\]

Sets of constraints \(2-14\)
5. Alternative mixed integer linear programming model

Formulation [mixed integer nonlinear programming model (MILP)]

Subject to (continued)

\[
q_{0j}^{t,r} - \beta_{0j}^{r} \varphi_{0j}^{t,r} \leq 0, \quad j \in J, t \in T_{0j}, r \in R_{0j}, \tag{26}
\]

\[
q_{0j}^{t,r} - \beta_{0j}^{r-1} \varphi_{0j}^{t,r} \geq 0, \quad j \in J, t \in T_{0j}, r \in R_{0j}, \tag{27}
\]

\[
y_{k}^{t,r} - \beta_{k}^{r} \varphi_{k}^{t,r} \leq 0, \quad k \in K_{D}, t \in T_{k}, r \in R_{k}, \tag{28}
\]

\[
y_{k}^{t,r} - \beta_{k}^{r-1} \varphi_{k}^{t,r} \geq 0, \quad k \in K_{D}, t \in T_{k}, r \in R_{k}, \tag{29}
\]

\[
\sum_{r \in R_{0j}} \varphi_{0j}^{t,r} \leq 1, \quad j \in J, t \in T_{0j}, \tag{30}
\]

\[
\sum_{r \in R_{k}} \varphi_{k}^{t,r} \leq 1, \quad k \in K_{D}, t \in T_{k}, \tag{31}
\]

\[
q_{0j}^{t,r} \geq 0, \quad \varphi_{1j}^{t,r} \in \{0, 1\} \quad j \in J, t \in T_{0j}, r \in R_{0j}, \tag{32}
\]

\[
y_{k}^{t,r} \geq 0, \quad \varphi_{k}^{t,r} \in \{0, 1\} \quad k \in K_{D}, t \in T_{k}, r \in R_{k}, \tag{33}
\]

Order quantities fall in the corresponding freight range

At most one freight range is active for each order quantity
6. Illustrative example

Data – 3 stage serial supply chain
(3 suppliers - 1 manufacturer – 1 Distribution Center, 6 time periods)

Table 2
Supplier characteristics.

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Capacity (units/period)</th>
<th>Ordering cost</th>
<th>Perfect rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Fixed ($)</td>
<td>Variable ($/Unit)</td>
</tr>
<tr>
<td>1</td>
<td>1200</td>
<td>4000</td>
<td>38</td>
</tr>
<tr>
<td>2</td>
<td>1300</td>
<td>4500</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>1100</td>
<td>5800</td>
<td>42</td>
</tr>
</tbody>
</table>

Table 3
Summary of parameters for the illustrative example.

<table>
<thead>
<tr>
<th>Period</th>
<th>Demand (units/period)</th>
<th>Production capacity (units/period)</th>
<th>Production cost Fixed ($)</th>
<th>Variable ($/Unit)</th>
<th>DC cost ($/order)</th>
<th>Unit holding cost ($/unit/period)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>900</td>
<td>1500</td>
<td>3000</td>
<td>12</td>
<td>2000</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>950</td>
<td>1500</td>
<td>3000</td>
<td>12</td>
<td>2000</td>
<td>3</td>
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<td>1100</td>
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<td>12</td>
<td>4400</td>
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</table>
6. Sensitivity analysis

GAMS 21.7 on a Pentium 4 with 3.4Ghz for MINLP – DICOPT algorithm in a package CPLEX 11.0 for MINLP(Maximum allowable gap of 0.001% between LP relaxation and optimal)

Table 5
Comparison of optimal solutions according to the MILP model, power and quadratic estimate functions.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Deviation from original cost value (%)</th>
<th>Total cost ($)</th>
<th>Gap (%)</th>
<th>Transportation cost error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Holding</td>
<td>Fixed production</td>
<td>Fixed DC</td>
<td>MILP</td>
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<tr>
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6. Sensitivity analysis

GAMS 21.7 on a Pentium 4 with 3.4Ghz for MINLP – DICOPT algorithm in a package CPLEX 11.0 for MINLP(Maximum allowable gap of 0.001% between LP relaxation and optimal)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Deviation from original cost value (%)</th>
<th>MILP</th>
<th>Power</th>
<th>Quadratic</th>
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<tr>
<td></td>
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<td>Fixed production</td>
<td>Fixed DC</td>
<td>(a) Supplier 1</td>
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</table>

(a) Number of periods, where raw material was ordered from one or more suppliers.
(b) Number of periods, where raw material was ordered from the specific supplier.
6. Conclusion

Overview

- A dynamic inventory model with supplier selection in a serial supply chain structure
- Capacity restriction for all stages
- Multiple level warehouse system
- Lead-time considered
- Formulation is developed and Analysis for transportation cost

Further research

- Multiple-product structure
- Joint replenishment and transportation costs
- Considering that uncertainty is a common factor in most of the actual production environment