An efficient heuristic for a two-stage assembly scheduling problem with batch setup times to minimize makespan

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Contents

Introduction

- Problem review
  - Two-stage assembly scheduling
  - Problem originated form motor factory
  - Batch setup time: starting processing components & switching the item of component
  - Product: each product is assembled with one or more common components

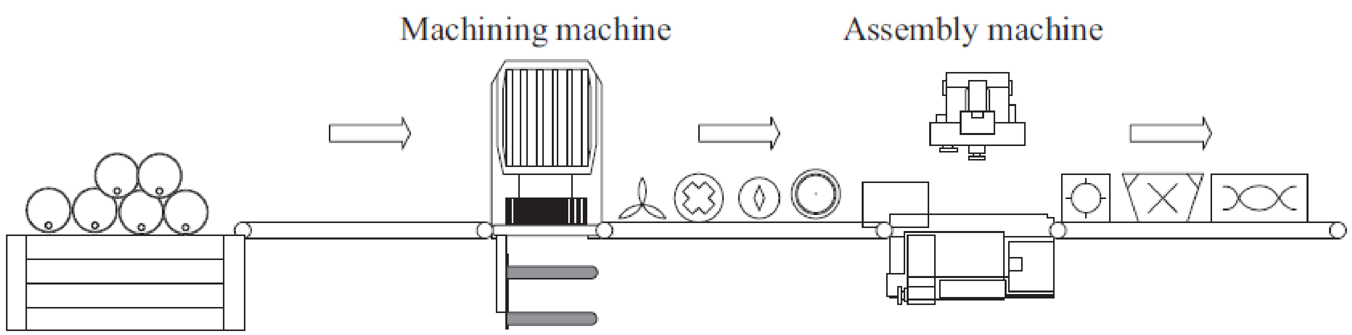

Fig. 1. Production flow line of a motor factory.

Introduction

- Example
  - ✓ A four-product and three-component type problem

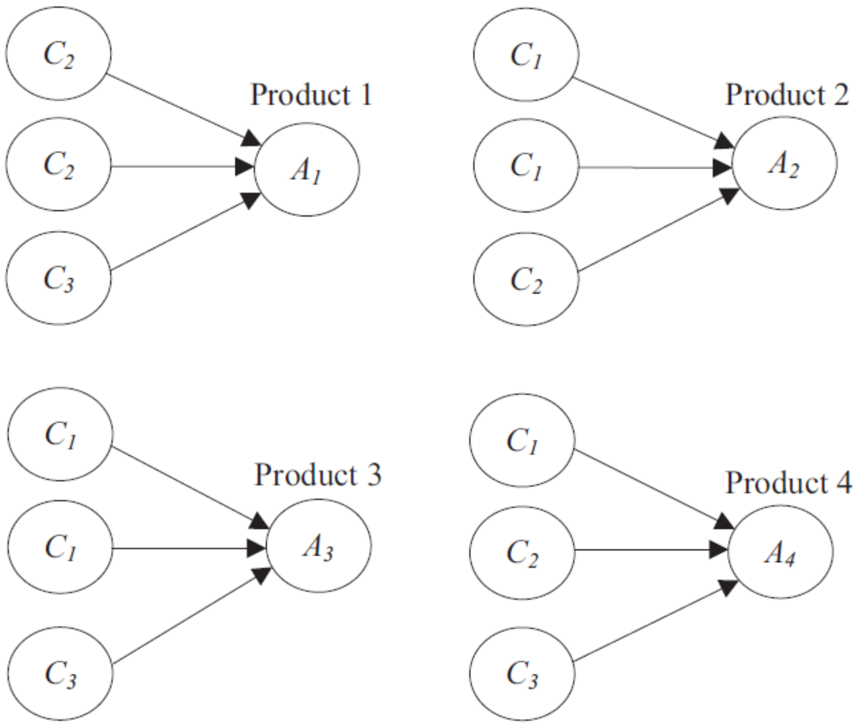

Fig. 2. An example of a four-product and three-component type.

Literature review

- *M*-machine flow shop scheduling
- Permutation flowshop problem
- Np-hard problem

- Literature review

Problem	Description	Authors
Single-machine extension	B&B algorithms to solve a problem with precedence constraints	Sidney and Potts (1975)
	weighted flow-time problems	Bansal [6] and Potts(1980)
	included job classes and setup times in their model	Mason and Anderson(1991)
Two-machines flowshop	B&B algorithm for the two-machine case	Ignall and Schrage(1965)
	Low bound based on the Lagrangean relaxation method	Van de Velde(1990)
M-machines flowshop	makespan objective	Potts(1980)
	total completion times(***compared***)	Ahmadi and Bagchi(1990)

Problem definition

- **Problem**
  - The two-stage assembly scheduling problem
- **Objective**
  - Minimum makespan
- **Decision Variable**
  - Sequencing
- **Approach & Algorithms**
  - Proposed heuristic
- **Assumption**
  - All components are available at time zero
  - At any time, machine can process at most one operation
  - non-preemptive
  - All setup times are identical
  - unlimited buffer
  - The processing constraint is non-permutation

Problem definition

- **Notations**
  - N number of products
  - n number of components types
  - L number of components for each product
  - M_i machine *i* , *i*=1,2
  - J_j product *j* , *j*=1,2,…,*N*
  - C_k component *k* , *k*=1,2,…,*n*
  - $t(C_k)$ processing time of C_k
  - A_j assembly operation of J_j
  - $t(A_j)$ operation time of A_j
  - s setup time
  - U set of unscheduled products
  - S set of scheduled products
  - B set of unscheduled components
  - D set of scheduled components
  - B_j set of unscheduled components in J_j
  - $|B|$ number of components types in *B*
  - $|B_j|$ number of components types in B_j
  - $T(B)$ total processing time of *B*
  - $T(B_j)$ total processing time of B_j

Problem definition

- Mixed integer programing (MIP):

  ❖ An optimal way is Formulate the problem into a mathematical problem, solve problem by commercial optimization software(CPLEX and Lingo)

  - Mixed integer programing (MIP):

Decision variables: $X_{k,l} = \begin{cases} 1, & \text{component } k \text{ is processed at position } l \text{ in the first stage} \\ 0, & \text{otherwise} \end{cases}$

$$Y_{j,p} = \begin{cases} 1, & \text{component } j \text{ is processed at position } p \text{ in the second stage} \\ 0, & \text{otherwise} \end{cases}$$

$$T_l = \begin{cases} 1, & \text{the components at position } l \text{ and } l-1 \text{ are the same in the first stage} \\ 0, & \text{otherwise} \end{cases}$$

Problem definition

Pos_l	type of components at position l in the first stage
$F_{1,l}$	finishing time of the component at position l in the first stage
$F_{2,p}$	finishing time of the product at position p in the second stage
Avt_j	available time of product j in the second stage

- Mixed integer programing (MIP)(con't):

Minimize $Z = C_{\max} = F_{2,N}$ (1)

subject to

$$\sum_{l=1}^{n} X_{k,l} = 1, \quad k = 1, \ldots, n \quad (2)$$

$$\sum_{k=1}^{n} X_{k,l} = 1, \quad l = 1, \ldots, n \quad (3)$$

M_1 can process at most one component at a time

$$\sum_{p=1}^{N} Y_{j,p} = 1, \quad j = 1, \ldots, N \quad (4)$$

$$\sum_{j=1}^{N} Y_{j,p} = 1, \quad p = 1, \ldots, N \quad (5)$$

M_2 can process at most one operation at a time

$$\mathrm{Pos}_l = \sum_{k=1}^{n} k \times X_{k,l}, \quad l = 1, \ldots, n \quad (6)$$

Pos_l type of components at position l in the first stage

$$T_l = \min\{|\mathrm{Pos}_l - \mathrm{Pos}_{l-1}|, 1\}, \quad \forall l \geqslant 2 \quad (7)$$

The components at position l and $l-1$ are same in first stage

$$F_{1,1} - s - \sum_{k=1}^{n} t(C_k) \times X_{k,1} \geqslant 0 \quad (8)$$

$$F_{1,l} - F_{1,l-1} - s \times T_l - \sum_{k=1}^{n} t(C_k) \times X_{k,l} \geqslant 0, \quad \forall l \geqslant 2 \quad (9)$$

Determine the completion time for each component

$$\mathrm{Avt}_j = \max\{\text{finishing time for each component in product } j\} \quad (10)$$

Available time of product j in the second stage

$$F_{2,1} - \sum_{j=1}^{N} t(A_j) \times Y_{j,1} \geqslant \sum_{j=1}^{N} \mathrm{Avt}_j \times Y_{j,1} \quad (11)$$

$$F_{2,p} - \sum_{j=1}^{N} t(A_j) \times Y_{j,p} \geqslant \max\left\{F_{2,p-1}, \sum_{j=1}^{N} \mathrm{Avt}_j \times Y_{j,p}\right\}, \quad \forall p \geqslant 2 \quad (12)$$

Determine the completion time for each product

Problem definition

- Mixed integer programing (MIP)(con't):

❖ Make sure model is linear:

$$T_l = \min\{|\mathrm{Pos}_l - \mathrm{Pos}_{l-1}|, 1\}, \quad \forall l \geqslant 2 \tag{7}$$

$$T_l \leqslant 1, \quad \forall l \geqslant 2 \tag{7.1}$$

$$T_l \leqslant \mathrm{Pos}_l - \mathrm{Pos}_{l-1} + M \times y_l, \quad \forall l \geqslant 2 \tag{7.2}$$

$$T_l \leqslant \mathrm{Pos}_{l-1} - \mathrm{Pos}_l + M \times (1 - y_l), \quad \forall l \geqslant 2 \tag{7.3}$$

where

$$y_l = \begin{cases} 1 & \mathrm{Pos}_l - \mathrm{Pos}_{l-1} \geqslant 0 \\ 0 & \mathrm{Pos}_l - \mathrm{Pos}_{l-1} < 0 \end{cases}$$

Computational experiments

Computing on:

- 1700MHz Pentium 4 processor under windows 2000
- Coded in VC ++ 5.0

Data:

- Six *p*-types(next slide)
- Eleven combinations of *m* and *n* values: (m; n)=(2,10), (4,10), (6,10), (8,10), (10,10),(2,15), (4,15), (6,15), (8,15), (2,20), and (4,20).
- Each case for 50 random problems

job processing time range:

- Discrete uniform distribution on $[a_{ik}, b_{ik}]$

Computational experiments

Table 6
Weighted problems: mean and standard deviation of node count, computation time, and % UB, and % stopped as a function of n and m

n/m	Node count		Time (s)		% UB		% stopped
	Mean	s.d.	Mean	s.d.	Mean	s.d.	
10/2	230.9	306.4	0.002	0.005	12.2	5.5	None
10/6	325.0	355.6	0.010	0.012	5.8	3.1	None
10/10	344.9	444.9	0.022	0.025	3.8	2.0	None
15/2	31,995.9	89,140.9	0.471	1.133	15.3	5.5	None
15/4	56,871.7	138,923.4	1.984	4.685	11.4	3.9	None
15/6	58,601.9	13,007.3	2.976	6.227	9.1	3.1	None
15/8	63,134.0	143,223.0	4.684	9.989	7.3	2.7	None
20/2	1,789,465.4	1,560,441.7	41.153	35.445	15.5	5.9	27.33
20/4	2,123,175.0	1,459,539.9	113.150	75.914	13.4	4.1	36.33

- Weight w_j follow a discrete uniform distribution[1,10]
- With n and m averaged over all p-type values
- Weighted problems are harder to solve and have higher UB values than unweighted ones.

Conclusion

- ***M*-machine permutation flow shop scheduling**
  - Total flow-time objective
  - Unweight and weighted version

- **Suggested**
  - A new machine-based lower bound
  - Dominance test

- **Future research**
  - The application of other solution techniques to the problem
  - Extending B&B algorithm to other objectives
  - The develop of efficient heuristics
  - Developing for big size problem

- **Adv & Disadv**

THANK YOU