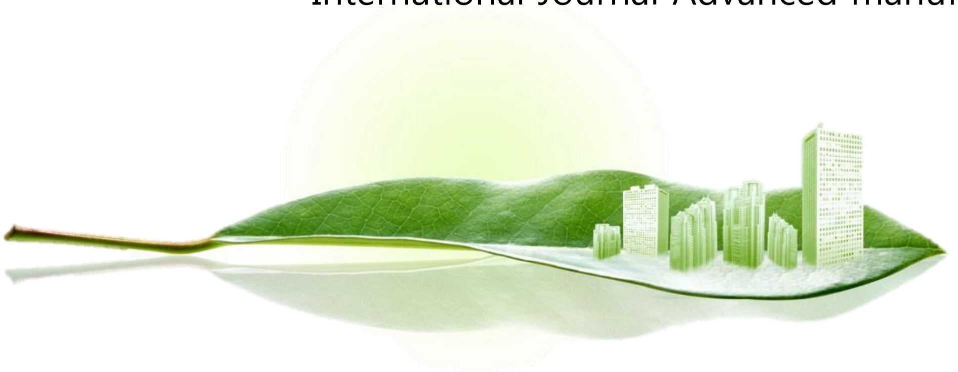
Two-stage flow-shop scheduling problem with non-identical second stage assembly machines

International Journal Advanced manufacturing Technology(2013)

J. Navaei et al



Kim Jong Min

2015.01.21

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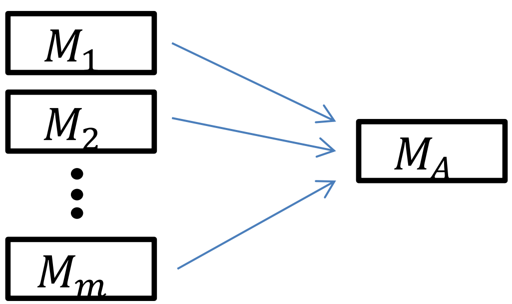
1. Introduction

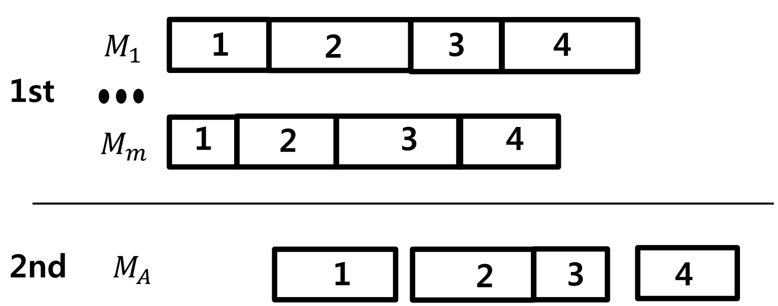
❖ Problem preview

➢ Model

✓ Two-stage assembly flow shop(TSAFP)

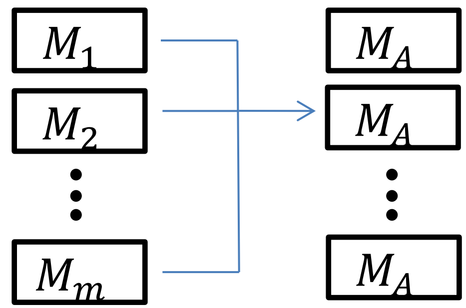
- 1^{st} stage : m machines
- 2^{nd}stage : single assembly machine

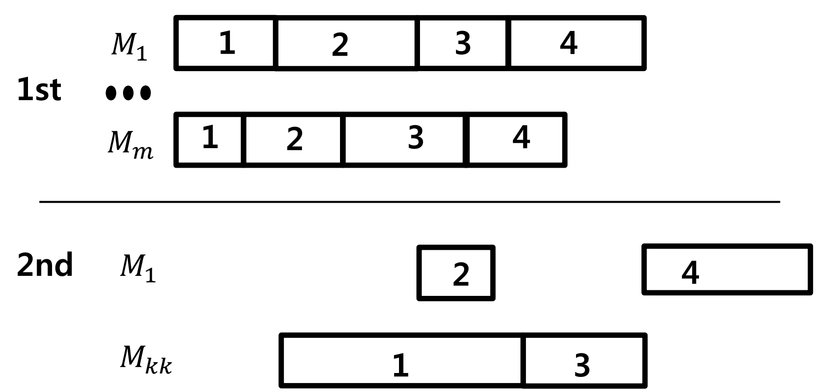




✓ Two-stage assembly flow shop(TSAFP)
with multiple non-identical assembly machines

- 1^{st} stage : m machines
- 2^{nd}stage : m assembly machines





1. Introduction

❖ Literature review

<table>
<tr><th>Type of stage</th><th>Author</th><th>Measure</th><th>Algorithm</th></tr>
<tr><td rowspan="6">Two-stage flow shop</td><td>Lee et al(1993)</td><td rowspan="5">Makespan</td><td rowspan="3">B&B</td></tr>
<tr><td>Potts et al(1995)</td></tr>
<tr><td>Hariri and potts(1997)</td></tr>
<tr><td>Al-Anzi et al(2006)</td><td>SA, TS, H-TS</td></tr>
<tr><td>Al-Anzi et al(2006) (Setup times)</td><td>TA, PSO</td></tr>
<tr><td>Al-Anzi et al(2005)</td><td>Makespan and mean completion time</td><td>ACO, SA, SDE</td></tr>
</table>

➢ There are several assembly machines at the second stage

2. Problem description

- Problem
  - ✓ TSAFP with multiple non-identical assembly machines
- Objective
  - ✓ To minimize makespan
- Decision variable
  - ✓ Sequence of job on each stage
  - ✓ Allocate to which machines at second stage
- Assumptions
  - ✓ Zero ready time
  - ✓ Preemption is not allowed
  - ✓ Each machine can process just one job at a time
- Constraints
  - ✓ The last operation at Stage 2 may start only after all m operations at Stage 1 are completed
- Solution approach
  - ✓ Hybrid Simulated annealing(SA)

2. Problem description

Job (i)
Position (j)
1st stage machine (b)
2nd stage machine (q)

❖ Notations

- T_{ib} Operation time of the job i on machine b at the first stage; $i = 1, \dots, \mathrm{n},\ b = 1, \dots, m$
- TT_{jb} Operation time of the job in position j in the first stage on machine b at the first stage; $j = 1, \dots, \mathrm{n}$
- P_{qi} Assembly time of the job i on assembly machine q at the second stage; $q = 1, \dots, \mathrm{kk}$
- P'_{qj} Assembly time of the job in position j in the first stage on assembly machine q at the second stage
- S_{qj} Completion time of the job in position j in the first stage on assembly machine q at the second stage
- FT_{jb} Completion time of an operation of job in position j which has to be processed on machine b in the first stage
- C_j Completion time of job in position j in the first stage, i.e., ready time of the job for the second stage
- CT_{qj} The completion time of the last job before job in position j which has been assigned on assembly machine q
- SS_q Completion time of the last job assembled on assembly machine q
- ZZ_{qj} The time which assembly process on job in position j can be started on machine q
- W Makespan

2. Problem description

❖ Notations

- $CT_{qj} = \max_{k=2}^{j} \{S_{q(k-1)}\} \quad \forall q = 1, \dots, kk, \quad \forall j = 2, \dots, n$
- $ZZ_{qj} = \max\left\{ C_j, \max_{k=2}^{j} \{S_{q(k-1)}\} \right\} \quad \forall q = 1, \dots, kk, \quad \forall j = 2, \dots, n$
- $C_j = \max_{b=1,2,\dots,m}\{FT_{jb}\} \quad \forall j = 1, \dots, n$
- $x_{ij} = \begin{cases} 1 & \text{if } i\text{th job is assigned to } j\text{th sequence in stage 1} \\ 0 & \text{otherwise; } \forall i = 1, \dots, n, \forall j = 1, \dots, n \end{cases}$
- $y_{ij} = \begin{cases} 1 & \text{if the job in position } j \text{ in the stage 1 is assigned to machine } q \text{ in the stage 2} \\ 0 & \text{otherwise; } \forall q = 1, \dots, kk, \ \forall j = 1, \dots, n \end{cases}$
- $S_{qj} = y_{qj} * \left\{\max\{C_j, CT_{qj}\} + P'_{qj}\right\} \quad \forall q = 1, \dots, kk, \ \forall j = 2, \dots, n, \ \forall k = 2, \dots, j$
- $SS_q = \max_{j=1,2,\dots,n}\{S_{qj}\} \quad \forall q = 1, \dots, kk$
- $W = \max_{q=1}^{kk} \{SS_q\}$
- $Min\ Z = W$

2. Problem description

❖ Mathematical model

$$Min\ Z = W$$

s.t:

$$ZZ_{qj} \geq C_j \quad \forall q = 1, 2, \ldots, kk; \quad \forall j = 2, \ldots, n \tag{1}$$

$$ZZ_{qj} \geq CT_{qj} \quad \forall q = 1, 2, \ldots, kk; \quad \forall j = 2, \ldots, n \tag{2}$$

$$CT_{qj} \geq S_{q(k-1)} \quad \forall q = 1, 2, \ldots, kk; \quad \forall k = 2, 3, \ldots, j; \forall j = 2, \ldots, n \tag{3}$$

$$S_{q1} = y_{q1} * \left\{ C_1 + \mathrm{P}'_{\mathrm{q}1} \right\} \quad \forall q = 1, 2, \ldots, kk \tag{4}$$

$$S_{qj} = y_{qj} * \left\{ ZZ_{qj} + \mathrm{P}'_{\mathrm{qj}} \right\} \quad \forall q = 1, 2, \ldots, kk; \forall j = 2, \ldots, n \tag{5}$$

2. Problem description

$$dd_{qj} = y_{qj} * ZZ_{qj}$$
$$ee_{qj} = y_{qj} * P'_{qj}$$

❖ Mathematical model

$$S_{qj} = y_{qj} * \left\{ ZZ_{qj} + \mathrm{P}'_{\mathrm{qj}} \right\} \quad \forall q = 1, 2, \ldots, kk; \forall j = 2, \ldots, n \tag{5}$$

$$S_{qj} = dd_{qj} + ee_{qj} \tag{5-1}$$

$$\boxed{\begin{aligned} & dd_{qj} \geq M * y_{qj} + ZZ_{qj} - M && \text{(5-2)} \\ & dd_{qj} \leq ZZ_{qj} && \text{(5-3)} \\ & dd_{qj} \leq M * y_{qj} && \text{(5-4)} \end{aligned}}$$

$$\boxed{\begin{aligned} & ee_{qj} \geq M * y_{qj} + P'_{qj} - M && \text{(5-5)} \\ & ee_{qj} \leq P'_{qj} && \text{(5-6)} \\ & ee_{qj} \leq M * y_{qj} && \text{(5-7)} \end{aligned}}$$

$$dd_{qj}, ee_{qj} \geq 0 \tag{5-8}$$

2. Problem description

❖ Mathematical model

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \forall i = 1, 2, \ldots, n \tag{6}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \forall j = 1, 2, \ldots, n \tag{7}$$

$$\sum_{q=1}^{kk} y_{qj} = 1 \quad \forall j = 1, 2, \ldots, n \tag{8}$$

$$TT_{jb} = \sum_{i=1}^{n} \left(x_{ij} * T_{ib}\right) \quad \forall j = 1, 2, \ldots, n;\ \forall b = 1, 2, \ldots, m \tag{9}$$

$$P'_{qj} = \sum_{i=1}^{n} \left(x_{ij} * P_{qi}\right) \quad \forall j = 1, 2, \ldots, n;\ \forall q = 1, 2, \ldots, kk \tag{10}$$

2. Problem description

❖ Mathematical model

$$FT_{jb} = \sum_{a=1}^{j} TT_{ab} \quad \forall j = 1, 2, \ldots, n; \ \ \forall b = 1, 2, \ldots, m \tag{11}$$

$$C_j \geq FT_{jb} \quad \forall j = 1, 2, \ldots, n; \ \ \forall b = 1, 2, \ldots, m \tag{12}$$

$$SS_q \geq S_{qj} \ \ \forall q = 1, 2, \ldots, kk; \ \ \forall j = 1, 2, \ldots, n \tag{13}$$

$$W \geq SS_q \quad \forall q = 1, 2, \ldots, kk \tag{14}$$

$$\begin{gathered} ZZ_{qj}, \ CT_{qj}, \ W \ , \ Cj, \ S_{qj}, \ \mathrm{P}'_{\mathrm{qj}} \ , \ P_{qi}, \ TT_{jb}, \ T_{ib}, \ FT_{jb}, \ SS_q \geq 0 \\ \forall q = 1, 2, \ldots, kk; \ \ \forall i = 1, 2, \ldots, n; \ \ \forall j = 1, 2, \ldots, n; \ \ \forall b = 1, 2, \ldots, m \end{gathered} \tag{15}$$

$$x_{ij} = 0, 1 \quad y_{qj} = 0, 1 \quad \forall i = 1, 2, \ldots, n; \forall j = 1, 2, \ldots, n; \ \ \forall q = 1, 2, \ldots, kk \tag{16}$$

3. Algorithms

❖ Hybrid Simulated Annealing(hybrid SA)

Begin Heuristic

Set heuristic parameters: T_1 = starting temperature, T_2 = terminal temperature, a = cooling factor, n = number of iterations;

Generate a random sequence for the first stage and randomly allot jobs to machines in the second stage to create a primary solution. Calculate makespan *F1* for this solution;

While $T_1 \geq T_2$

Repeat1 n times

Pick two random positions i and j from the first stage sequence and swap; calculate ready times C for current solution;

Create vector S of all zeros which shows the time that assembly machines become idle for each allocation;

Assembly machine selection heuristic →

Begin proposed heuristic

Calculate finishing time of job in position j on assembly machine q *for* all combinations and form matrix M;

Repeat2 till all jobs are allocated to assembly machines:

Find the minimum value(s) of the matrix M;

Randomly select one of the minimum values and allot job j *to* assembly machine q for which the matrix M value is minimum;

Update vector S;

Remove the selected job (row) from further matrices and update matrix M considering vector S;

End repeat2

Calculate makespan for the current sequence of jobs and allocation of jobs on assembly machines using S: F2← max{S};

End proposed heuristic

If F2<F1 then F1←F2,

If F2>F1 Then accept the solution with probability p and set F1←F2, where;

$$d = \left| \frac{F2 - F1}{F1} \right|$$

$$p = e^{-\left(\frac{d}{T1}\right)}$$

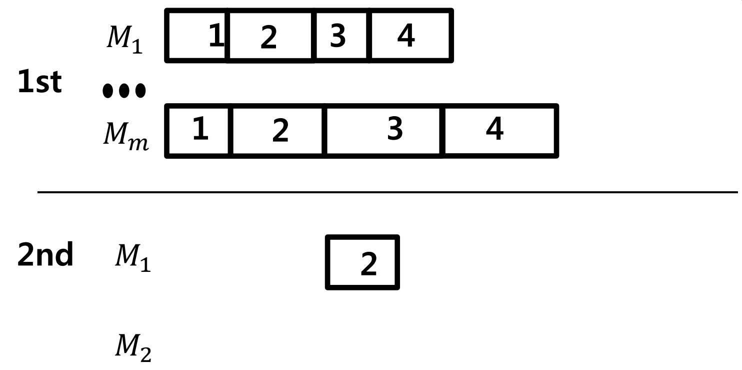
End Repeat1

Set $T_1 = T_1 * a$;

End While

End Heuristic

3. Algorithms

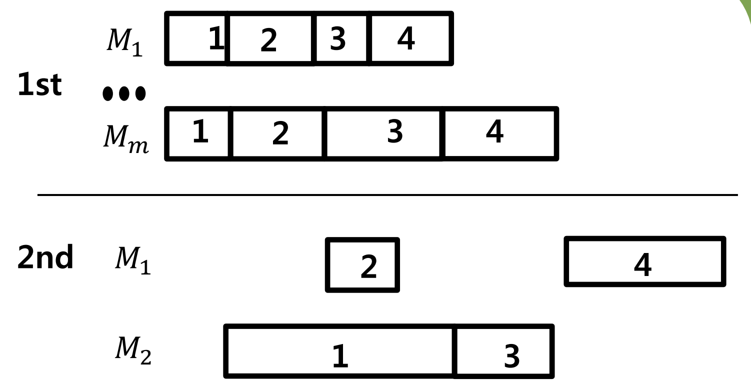


❖ Hybrid Simulated Annealing(hybrid SA)

- Assembly machine selection heuristic
  - ✓ Ex) Job : 4,
    2nd stage assembly machine : 2

1. Ready time & Matrix PP(assembly time of jobs on assembly machines)
   - $C_1 = 2, C_2 = 5, C_3 = 9, C_4 = 13$
   - $PP = \begin{bmatrix} 11 & 8 \\ 2 & 3 \\ 9 & 2 \\ 7 & 9 \end{bmatrix}$
2. Vector S (the time which assembly machines become idle for each allocation)
   - $S = [0 \quad 0]$
3. Matrix M
   - $M^1 = \begin{bmatrix} 13 & 10 \\ 7 & 8 \\ 18 & 11 \\ 20 & 22 \end{bmatrix}$ $\quad S_{q1} = y_{q1} * \left\{C_1 + \mathrm{P}'_{q1}\right\} \quad \forall q = 1, 2, \ldots, kk$
4. Vector S updating
   - $S = [7 \quad 0]$

3. Algorithms



❖ Hybrid Simulated Annealing(hybrid SA)

➢ Assembly machine selection heuristic

$$S_{qj} = y_{qj} * \left\{ ZZ_{qj} + P'_{qi} \right\} \quad \forall q = 1, 2, \ldots, kk; \ \forall j = 2, \ldots, n$$

$$ZZ_{qj} = \max \left\{ C_j, \max_{k=2}^{j} \left\{ S_{q(k-1)} \right\} \right\}$$

5. Matrix M
   - $M_{1,1} = \max\{2,7\} + 11 = 18$
   - $M_{2,1} = \max\{9,7\} + 9 = 18$
   - $M_{3,1} = \max\{13,7\} + 7 = 20$
   - $M^2 = \begin{bmatrix} 18 & 10 \\ 18 & 11 \\ 20 & 22 \end{bmatrix}$
6. Vector S updating
   - $S = [7 \quad 10]$
7. Matrix M
   - $M_{1,2} = \max\{9,10\} + 2 = 12$
   - $M_{2,2} = \max\{13,10\} + 9 = 22$
   - $M^3 = \begin{bmatrix} 18 & 12 \\ 20 & 22 \end{bmatrix}$
8. Vector S updating
   - $S = [7 \quad 22]$
9. Matrix M
   - $M_{1,2} = \max\{13,22\} + 9 = 31$
   - $M^4 = [20 \quad 31]$

3. Algorithms

❖ Hybrid Simulated Annealing(hybrid SA)

➢ Assembly machine selection heuristic

✓ In more than one minimum case

Wise selection

- A single machine : $M = \begin{bmatrix} 13 & 10 \\ 7 & 11 \\ 7 & 8 \\ 20 & 22 \end{bmatrix}$ $M = \begin{bmatrix} 13 & 10 \\ 7 & 11 \\ 7 & 8 \\ 20 & 22 \end{bmatrix}$
- A single job : $M = \begin{bmatrix} 13 & 10 & 15 \\ 7 & 7 & 12 \\ 18 & 8 & 14 \\ 20 & 22 & 16 \end{bmatrix}$ $M = \begin{bmatrix} 13 & 10 & 15 \\ 7 & 7 & 12 \\ 18 & 8 & 14 \\ 20 & 22 & 16 \end{bmatrix}$

Random selection

- Random : $M = \begin{bmatrix} 13 & 10 \\ 7 & 11 \\ 7 & 8 \\ 20 & 22 \end{bmatrix}$ $M = \begin{bmatrix} 13 & 10 \\ 7 & 11 \\ 7 & 8 \\ 20 & 22 \end{bmatrix}$

3. Algorithms

❖ Hybrid Simulated Annealing(hybrid SA)

✓ Initial sequences

- *Random*

✓ Neighborhood generation(Swap)

1	3	2	4

↓

1	4	2	3

✓ Neighborhood movement

- Probability $p = e^{-(\frac{d}{T1})}$, $d = \left|\frac{F2-F1}{F1}\right|$

✓ Termination condition

- Stop when the finial temperature reaches to the steady state

✓ Parameter setting

- **The initial temperature (T_1) :**
  - 0.01, 0.1, 10, 100, 1000 and 10000
- **The final temperatures (T_2) :**
  - 0.005, 0.001, 0.0005, 0.0001, 0.00005 and 0.00001
- **The cooling factor** (a):
  - 0.94, 0.95, 0.96, 0.97, 0.98 and 0.99
- **The number of iterations** (noi):
  - 10, 20, 40, 60, 80, 100, 120, 150 and 200

◆ Best results parameter setting

- **The initial temperature (T_1)** : 10
- **The final temperatures (T_2)** : 0.0001
- **The cooling factor** (a): 0.96
- **The number of iterations** (noi): 100

4. Computation result

❖ Experimentation environment

- Number of jobs : 30, 50, 70, 90 and 110
- Number of machines : - first stage(m) : 2, 4, 6 and 8
  - second stage(kk) : 2, 3 and 4
- Uniform distribution : - first stage(m) : $[0,100]$
  - second stage(kk) : $[1,100]$
- Comparison the performance of the heuristics using measure :
  - Average error solution error
  - Average best solution error
  - Average worst solution error
  - Coefficient of variation(CV)
  - Run time

4. Computation result

Table 1 Summary of computational results of the mathematical model and the heuristic

n	*m*	*kk*	Model	Heuristic						
			Run time	Average mean solution error (%)	Average best solution error (%)	Average worst solution error (%)	Coefficient of variation	Run time	No. of better solutions	No. of same solutions
30	2	2	15.7	2.05	0.02	5.15	0.013	2.0	–	–
		3	30.1	1.11	0.02	3.32	0.010	3.3	–	–
		4	37.8	1.12	0.04	2.57	0.008	4.0	–	–
	4	2	29.1	1.59	0.02	4.66	0.012	2.0	–	–
		3	43.7	1.17	0.00	3.85	0.011	3.5	–	–
		4	65.5	1.02	0.02	3.09	0.009	4.1	–	–
	6	2	57.0	1.99	0.04	4.55	0.013	2.1	–	–
		3	66.6	1.18	0.00	3.40	0.011	3.6	–	–
		4	77.0	1.12	0.02	3.11	0.009	4.1	–	–
	8	2	69.1	1.72	0.04	4.86	0.013	2.1	–	–
		3	81.1	1.39	0.06	4.29	0.012	3.4	–	–
		4	170.3	0.80	0.00	2.92	0.008	4.1	–	–
50	2	2	41.4	1.33	0.20	3.42	0.009	5.2	–	–
		3	141.9	0.95	0.07	2.72	0.007	8.7	–	–
		4	168.5	0.89	0.10	2.26	0.006	9.4	–	–
	4	2	128.3	1.1	0.04	2.92	0.008	5.3	–	–
		3	278.2	0.82	0.01	2.14	0.007	9.0	–	–
		4	461.3	0.58	0.00	1.88	0.005	10.2	–	–
	6	2	244.1	1.06	0.03	2.76	0.008	5.6	–	–
		3	470.8	0.86	0.09	2.37	0.007	9.3	–	–
		4	696.5	0.77	0.00	2.05	0.006	10.3	–	–
	8	2	316.6	1.19	0.06	2.93	0.009	5.7	–	–
		3	895.3	0.98	0.06	2.74	0.007	9.6	–	–
		4	1,660.6	0.49	0.04	1.48	0.004	10.3	–	–

4. Computation result

70	2	2	162.6	0.93	0.04	2.14	0.007	11.5	–	–
		3	389.1	0.74	0.06	1.92	0.006	16.6	–	–
		4	659.8	0.50	0.03	1.34	0.004	19.4	–	–
	4	2	519.7	0.77	0.04	1.98	0.006	11.7	–	–
		3	1,072.2	0.68	0.07	1.55	0.004	16.6	–	–
		4	1,406.1	0.51	0.05	1.70	0.004	20.6	–	–
	6	2	686.7	0.79	0.08	2.11	0.005	12.1	–	–
		3	1,830.2	0.58	0.07	1.40	0.004	17.7	–	–
		4	2,448.2	0.39	0.00	1.09	0.003	20.5	–	–
	8	2	1,514.7	0.91	0.00	2.26	0.006	12.2	–	–
		3	2,910.4	0.59	0.06	1.27	0.004	17.9	–	–
		4	3,171.4	0.36	0.00	1.08	0.003	20.7	–	–
90	2	2	621.4	0.79	0.03	1.96	0.006	18.8	–	–
		3	1,319.7	0.47	0.04	1.36	0.004	27.3	–	–
		4	1,780.5	0.44	0.01	1.32	0.004	32.1	–	–
	4	2	1,254.1	0.61	0.04	1.92	0.005	18.8	–	–
		3	2,824.6	0.57	0.04	1.26	0.004	27.4	–	–
		4	3,600*	0.27	−0.06	0.96	0.003	31.2	2	1
	6	2	2,311.4	0.64	0.01	1.35	0.004	19.3	–	–
		3	3,600*	0.38	−0.08	1.30	0.004	28.0	2	0
		4	3,600*	0.40	−0.04	1.23	0.004	32.2	2	1
	8	2	3,600*	0.46	−0.15	1.84	0.005	19.5	3	1

4. Computation result

		3	3,600*	0.18	−0.42	1.09	0.004	28.7	4	1
		4	3,600*	0.13	−0.23	0.84	0.003	33.8	3	0
110	2	2	3,600*	0.47	0.00	1.44	0.004	26.4	2	1
		3	3,600*	0.34	0.00	1.12	0.003	38.5	2	2
		4	3,600*	0.31	0.01	0.80	0.002	45.2	1	3
	4	2	3,600*	0.34	0.00	1.14	0.004	26.7	1	2
		3	3,600*	0.28	−0.05	0.91	0.003	39.0	2	0
		4	3,600*	0.12	−0.12	0.50	0.002	45.4	3	2
	6	2	3,600*	0.24	−0.32	0.93	0.004	26.9	3	0
		3	3,600*	0.50	0.02	1.25	0.003	39.0	1	1
		4	3,600*	0.12	−0.29	0.99	0.004	45.4	3	1
	8	2	3,600*	0.49	0.00	1.20	0.004	25.9	2	2
		3	3,600*	−0.66	−0.91	0.17	0.003	41.3	5	0
		4	3,600*	0.02	−0.25	0.97	0.003	46.0	4	0

- 3600* : more than 3600 sec
- All of the problems are less than 2% in Average mean solution error
- Large problem with more than 50 jobs are less than 1% in Average mean solution error

4. Computation result

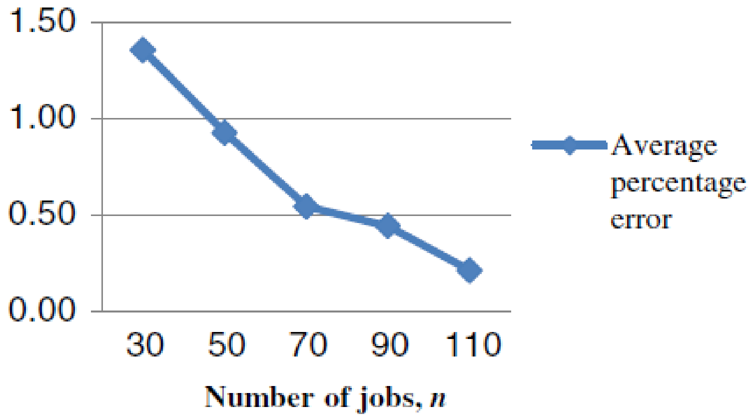


Fig. 1 Average percentage error for different number of jobs (n)

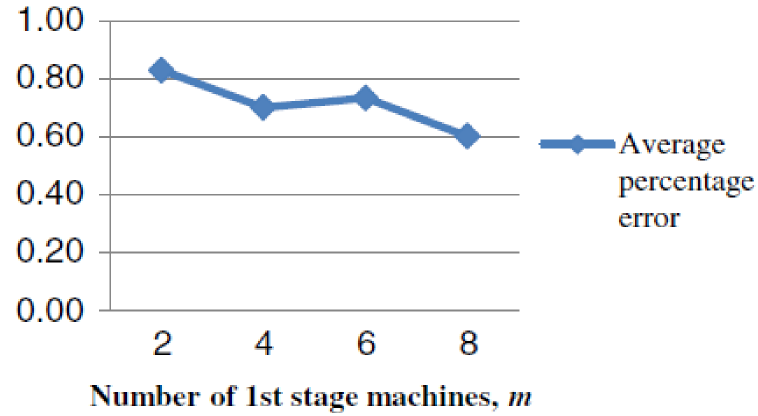


Fig. 2 Average percentage error for different number of first stage machines (m)

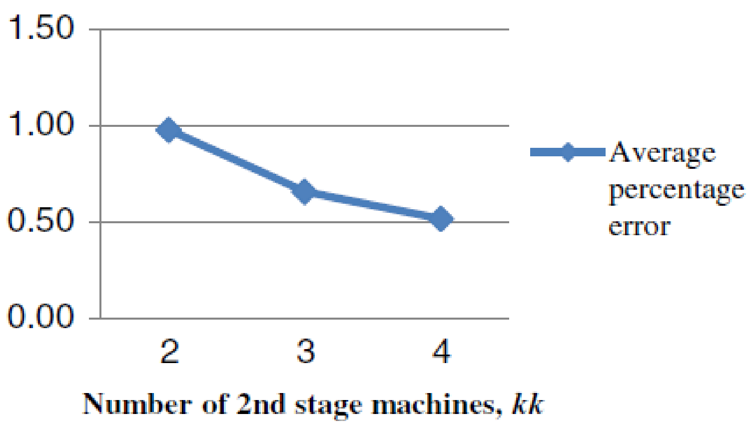


Fig. 3 Average percentage error for different number of second st machines (kk)

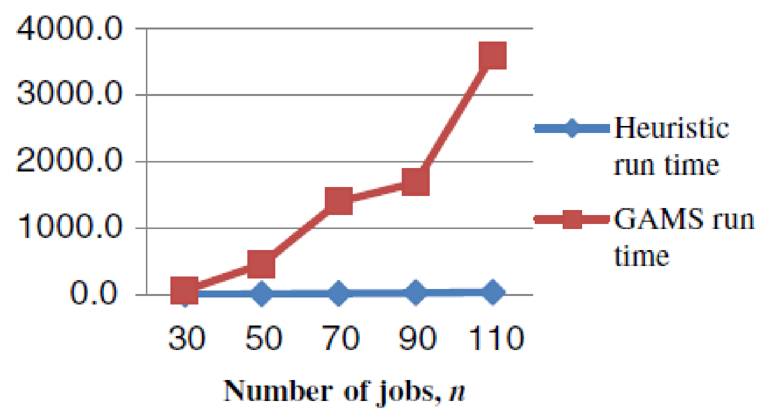


Fig. 4 Average run times of model and heuristic for different number of jobs (n)

4. Computation result

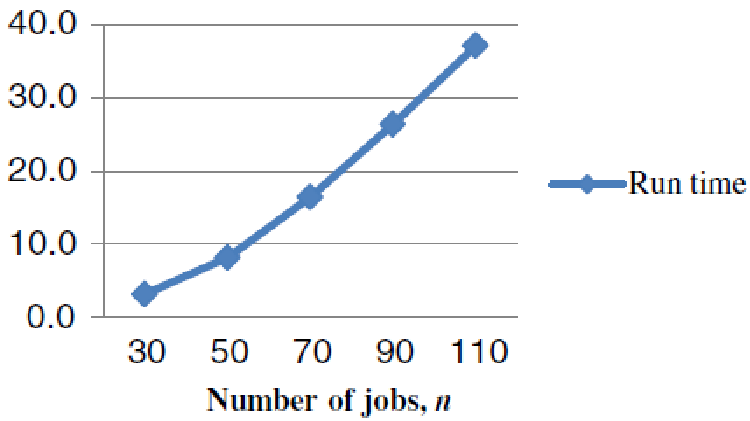


Fig. 5 Average run times for different number of jobs (*n*)

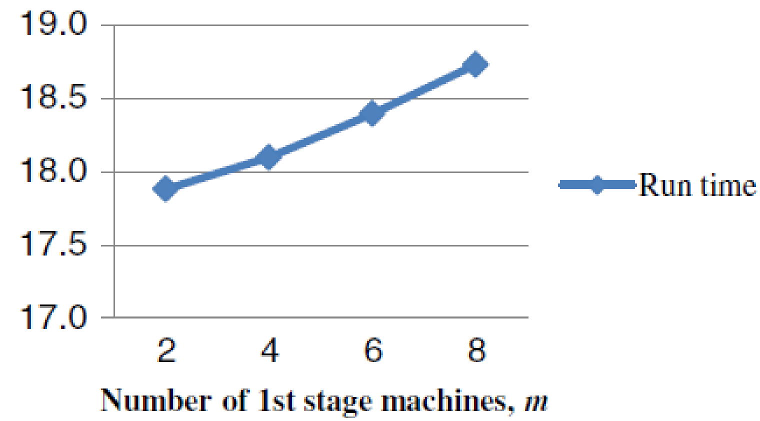


Fig. 6 Average run time for different number of first stage machines (*m*)

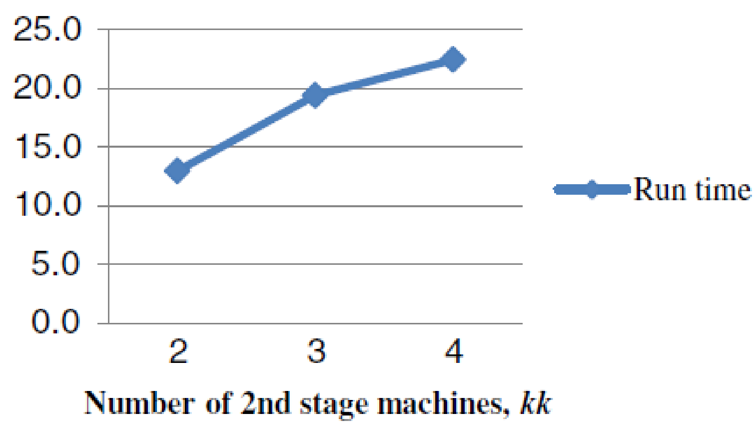


Fig. 7 Average run time for different number of second stage machines (*kk*)

4. Computation result

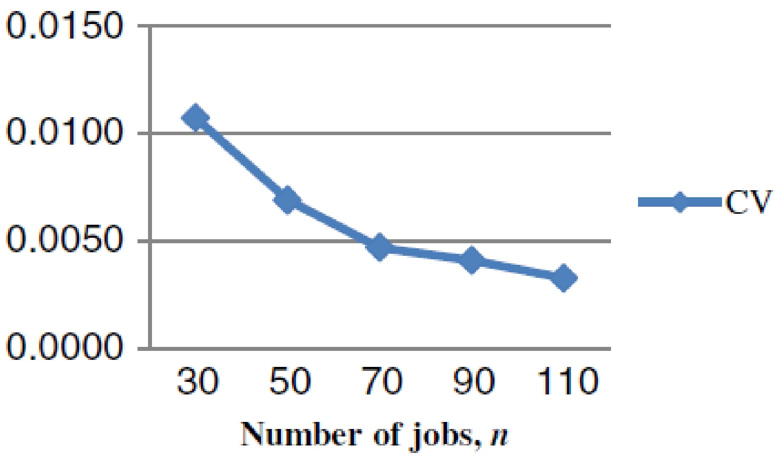


Fig. 8 Average CV for different number of jobs (n)

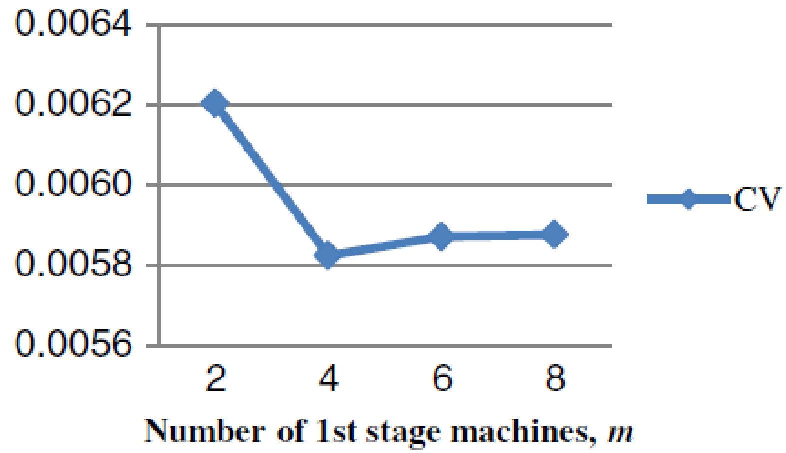


Fig. 9 Average CV for different number of first stage machines (m)

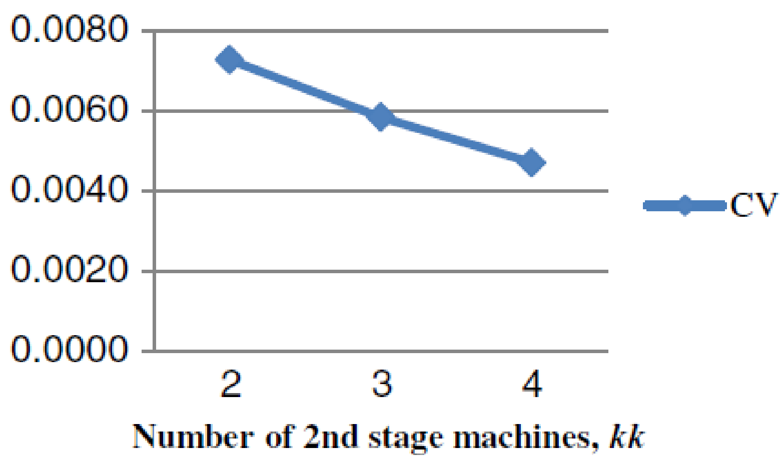


Fig. 10 Average CV for different number of second stage machines (kk)

4. Computation result

Table 2 Comparison between initial scheduling and hybrid SA heuristic

Production plan	n	m	kk	Initial makespan	Hybrid SA makespan	Improvement rate (%)
1	14	5	3	2,147.6	1,035	51.81
2	10			1,563.9	873	44.18
3	13			1,909.7	1,174	38.52
4	11			1,696.5	903	46.77
5	33			6,147.7	3,473	43.51
6	18			2,835.3	1,873	33.94
7	21			3,337.1	2,280	31.68
8	26			4,171.7	2,357	43.50
9	7			11,391.9	6,893	39.49
10	29			4,613.7	2,967	35.69
11	19			2,850.9	1,783	37.46
12	18			2,722.2	1,689	37.95
13	22			3,420.3	2,180	36.26
14	26			3,936.4	2,236	43.20
15	23			3,442.4	1,984	42.37
					Average improvement	40.42

- Experimentation Environment
  - Application : bedroom furniture
  - First stage (m) : 5
  - Second stage (kk) : 3

5. Conclusions

❖ Summary

- TSAFP with multiple non-identical assembly machines
- Makespan
- Hybrid SA

❖ Future research

- Other heuristics
- Other objectives
- Sequence-dependent setup times, Buffer between stages and Priorities in jobs

❖ Adv. and Dis-adv

- Adv.

- Dis-adv.