Analysis of ambulance decentralization in an urban emergency medical service using the hypercube queueing model

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Contents

Introduction

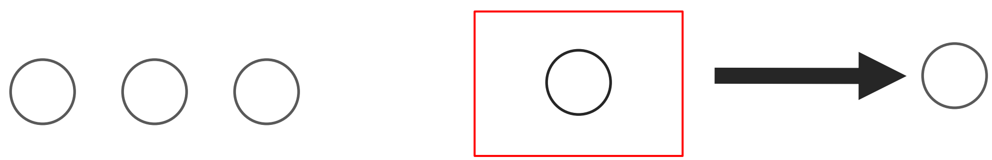
- Emergency Medical Services(EMS)
  - ✓ To rapidly provide first care medical assistance to the victims.
  - ✓ EMS system : Type and location of the request, number and location of ambulances, system congestion, local traffic conditions, weekday and time, etc.

- Objective
  - ✓ Analyzing the effects of decentralizing ambulances and adding new ambulances to the system, comparing the results to the ones of the original situation in Brazil.

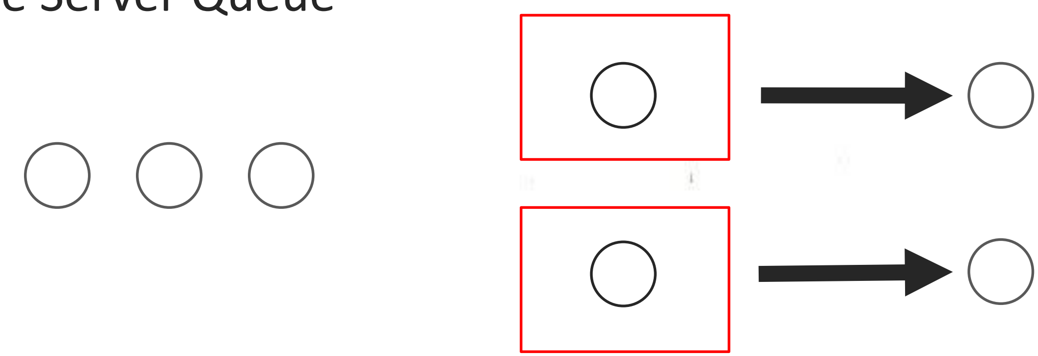
Introduction

- queueing model
  - ✓ The mathematical study of waiting line or queues.
  - ✓ Can predict queue lengths and waiting times
  - ✓ Generally considering a branch of operations research

  - ✓ Number of servers available
    - ➢ Single Server Queue



    - ➢ Multiple Server Queue



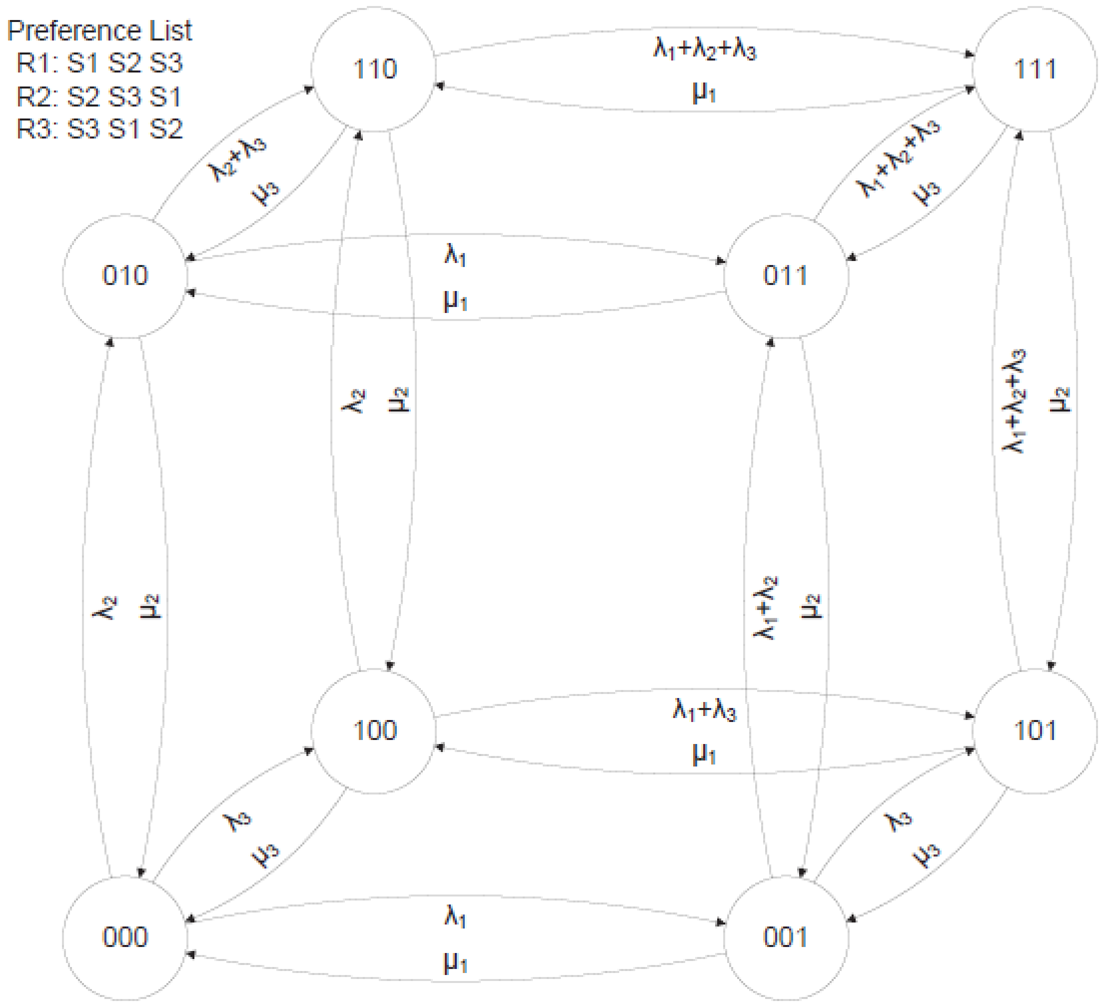
Introduction

- Hypercube queueing model
  - ✓ Developed by Larson
  - ✓ Effective descriptive model for planning server-to-custom
  - ✓ Having a hypercube structure when the number of servers is more than three.

  - ✓ Server i is available (0) or busy (1)
  - ✓ Each state is a number in base two.
  - ✓ Each digit shows the state of the corresponding server
  - ✓ Each region, calling atoms(*j*) existing a priority list of servers
  - ✓ Service requests in Poisson Distribution $\boldsymbol{\lambda}_j$
  - ✓ Distributed equal service rates μ_i

Introduction

- Hypercube queueing model



Literature Review

- [1]M.O. Ball, F.L. Lin(1993)

  A reliability model applied to emergency service vehicle location
- [2]M. Gendreau, G. Laporte, F. Semet(2001)

  A dynamic model and parallel Tabu search heuristic for real-time ambulance relocation

- [13]R.C. Larson(1974)

  A hypercube queueing model for facility location and redistricting in urban emergency services
- [21]R.C. Larson, A.R. Odoni(1981)

  Urban operations research
- [23]T.H. Burwell, J.P. Jarvis, M.A. McKnew(1993)

  Modeling co-located servers and dispatch ties in the hypercube model

SAMU-192

- SAMU(Service d'Aide Medicale Urgente)
  - ✓ SAMU's serve a city or a region containing several cities and are integrated into a group of public hospitals.
  - ✓ Having a phone center that receive all requests, teams of doctors, nurses and drivers, ambulances equipped for basic life support (basic support vehicles, BSVs), and units of advanced life support (advanced support vehicles ,ASVs).
- SAMU-192
  - ✓ The EMS of Campinas was reorganized yielding the SAMU-192
  - ✓ The urban regulator center of medical emergencies.
  - ✓ Providing a queueing service for incoming call if all ambulances were busy at the time of their requests
  - ✓ One of the special concerns of the system manager was that the ambulance workloads were too high in certain periods of the day, causing long waiting times for users.

SAMU-192

- To apply the hypercube model in SAMU-192
  - ✓ Two stages of data collection
    - ➢ Determining the type and amount of past services
    - ➢ A detailed sample was collected to verify the critical model assumptions

Table 1
Performance measures of SAMU-192

Measures	Peak		24 h	
	BSV	ASV	BSV	ASV
Arrivals/day	20	2	70	10
Inter-arrival time (min)	10	92	14	106
Response time (min)[a]	13	11	12	10
Time on scene (min)	37	39	37	39
Service time (min)	66	63	64	62

[a]Travel time + setup time.

- Based on 209 requests (190 basic and 19 advanced calls) of this sample that occurred during peak periods

SAMU-192

- SAMU-192(Service d’Aid Medical Urgent)

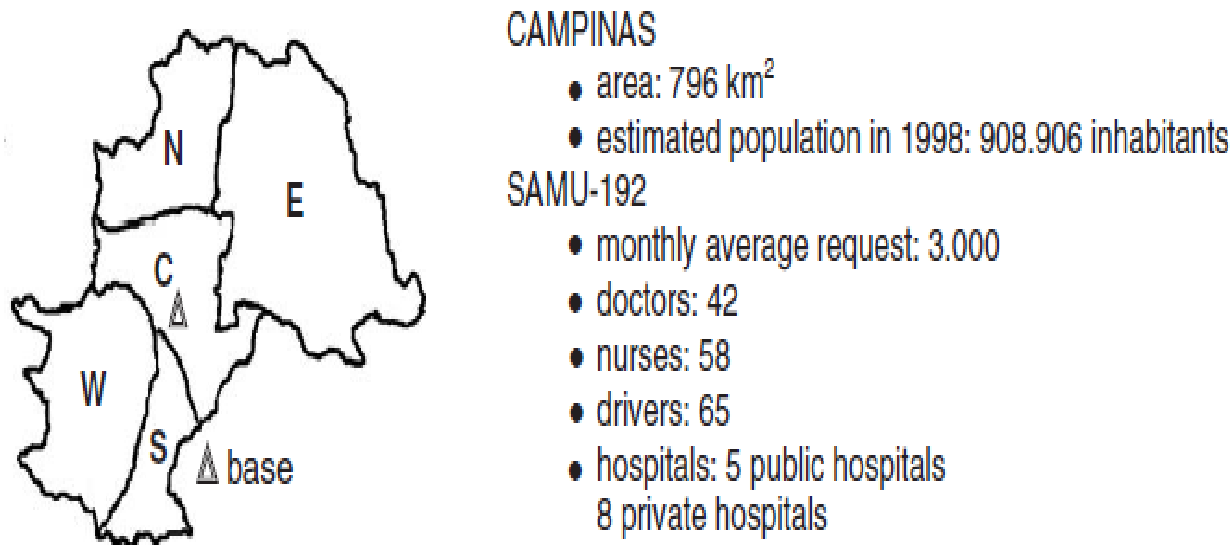


Fig. 1. Coverage area of SAMU-192.

Hypercube model

- Hypercube model
  - ✓ Considering geographical and temporal complexities of the region
  - ✓ Based on spatial queueing theory and Markovian analysis approximations.
  - ✓ To expand the state space description of a simple multi-server queueing system in order to represent each server individually
  - ✓ To incorporate more complex dispatching policies.
  - ✓ Estimating either region-wide or for each server or region, such as workloads, mean response times, fraction of dispatches of each server to each region

  - ✓ Describing the status of the ambulances: Free (0) , busy (1)

    ex) The state 011 (3-server system)

    –> ambulances 1 and 2 busy, and 3 free

Model assumption

- Assumptions for the SAMU-192 study
  1. Region divided in 5 areas, each area divided in 2 layers(layer for ASV, Layer for BSV)
  2. Emergency calls in Poisson Distribution ($\boldsymbol{\lambda}_j$, j=1…N)
  3. Travel time from i to j, estimated for each layer of each region. ((i, j=1,2,…,N_A))
  4. BSV = 8, ASV = 2, the central base = 1
  5. Servers locations are known.
  6. Service assignment was queued.
  7. Fixed-preference dispatching : There are a list of dispatching preferences for each atom.
  8. Service time is composed by the setup time, the travel time, the on-scene time, the travel time back to the base.
  9. Service rates : μ_n(n=1,2,….,N)
  10. Service time dependence on travel time

Model assumption

Table 2
Mean arrival rates of atoms

Atom	λ_j (calls/hour)
1 - *NB*	0.8535
2 - *NA*	0.1035
3 - *SB*	0.8535
4 - *SA*	0.0776
5 - *EB*	0.8535
6 - *EA*	0.0776
7 - *WB*	1.5001
8 - *WA*	0.1293
9 - *CB*	0.8276
10 - *CA*	0.1293
Total	5.4054

Model assumption

Table 3
Fixed-preference dispatching matrix (original configuration)

Atom	Preference dispatching									
	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
1 - *NB*	9	10	3	4	5	6	7	8	2	1
2 - *NA*	1	2	10	7	3	4	9	5	8	6
3 - *SB*	10	3	4	5	6	7	8	9	1	2
4 - *SA*	2	1	5	3	10	9	4	6	7	8
5 - *EB*	3	4	7	10	8	5	6	9	2	1
6 - *EA*	1	2	6	9	4	10	5	3	8	7
7 - *WB*	4	5	8	6	7	3	9	10	2	1
8 - *WA*	2	1	9	8	3	5	10	4	6	7
9 - *CB*	5	6	8	10	9	4	3	7	1	2
10 - *CA*	2	1	10	5	6	8	7	9	3	4
Vehicles										

Table 4
Mean service times and rates of ambulances

Ambulance	Service time[a]	Standard deviation	Coefficient of variation	μ_n[b]
1	61	40	0.7	0.9836
2	64	44	0.7	0.9375
3	63	52	0.8	0.9524
4	70	48	0.7	0.8571
5	61	44	0.7	0.9836
6	66	58	0.9	0.9091
7	64	50	0.8	0.9375
8	61	40	0.7	0.9836
9	70	58	0.8	0.8571
10	70	56	0.8	0.8571
ASV	63	42	0.7	0.9606
BSV	66	51	0.8	0.9172
Total	65	49	0.8	0.9259

[a]minutes,
[b]calls/hour.

Steady state probabilities

- The equilibrium probability equations
  - ✓ A vector with N= 10 elements (ambulances)
  - ✓ Values 0 or 1 (free or busy)
  - ✓ 2^{10}=1024 possible states
  - ✓ Defining the queue, namely $S_{11}, S_{12},\ldots,S_{20}$

ex) 1000000101 (ambulances 1,3 and 10 are busy)

$$
\begin{aligned}
&[(\lambda_2+\lambda_4+\lambda_6+\lambda_8+\lambda_{10})+(\lambda_7+\lambda_3+\lambda_5)+(\lambda_9)+(\lambda_1)+(\mu_1)+(\mu_3)+(\mu_{10})]\mathrm{p}_{1000000101}\\
&\quad=\mu_2\mathrm{p}_{1000000111}+\mu_4\mathrm{p}_{1000001101}+\mu_5\mathrm{p}_{1000010101}+\mu_6\mathrm{p}_{1000100101}+\mu_7\mathrm{p}_{1001000101}\\
&\qquad+\mu_8\mathrm{p}_{1010000101}+\mu_9\mathrm{p}_{1100000101}+(\lambda_2+\lambda_6)\mathrm{p}_{1000000100}\\
&\qquad+(\lambda_3+\lambda_5)\mathrm{p}_{1000000001}+\lambda_3\mathrm{p}_{0000000101}.
\end{aligned}
$$

Steady state probabilities

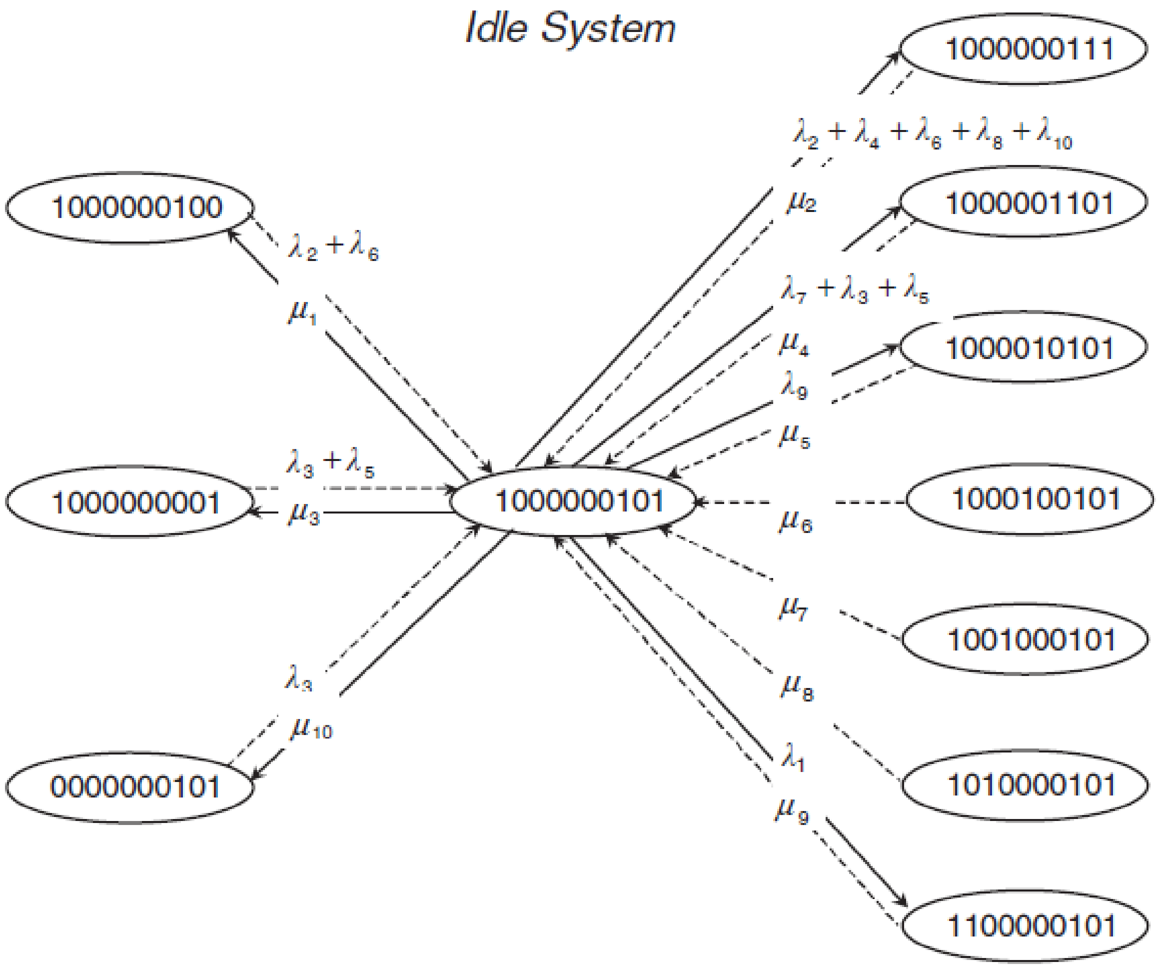


Fig. 2. Flows in and out of state 1000000101.

Performance measures

- A function of the equilibrium distribution

✓ Parameters

➢ $p_{0000000000} = 0.003$ The probabilities of finding the system empty and saturated

➢ $p_s = 0.09$ The system of saturation probability

✓ Decision variable

➢ p_n The work load of ambulance n represents the fraction of time that ambulance n is busy

p_n = A function (the sum of the equilibrium probabilities)

Performance measures

- Average travel times

✓ Parameters

- i Index for atom
- j Index for atom
- N The fraction of time
- τ_{ij} Matrix of travel times
- f_{nj} [1] The fraction of all dispatches that send ambulance n, when available, to atom j
- t_{nj} The mean time required to ambulance n

✓ Decision variable

- $\overline{T_Q}$ The mean travel time to a queued call

Performance measures

- Average travel times(Mathematical formulation)

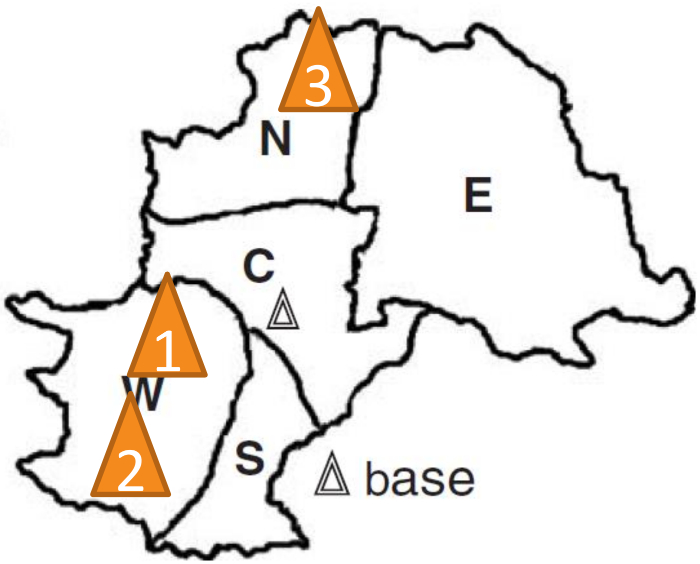
$$\bar{T} = \sum_{n=1}^{N} \sum_{j=1}^{N_A} f_{nj}{}^{[1]} \; t_{nj} + p_s \overline{T_Q}$$

- ✓ Resulted in $\bar{T}$ = 13.6 min
- ✓ ASVs = 10.9 min
- ✓ BSVs = 14.2 min
- ✓ Deviation of only 3.3% and 5.1%

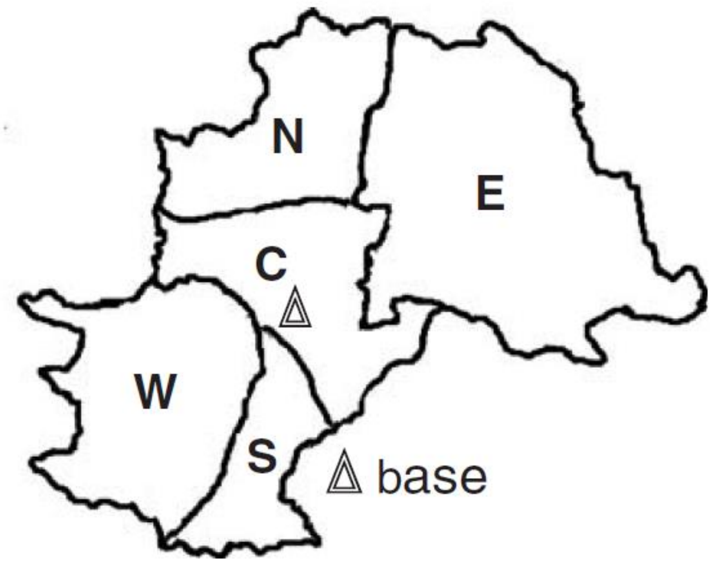
Evaluation of alternative scenarios

- Ambulance decentralization
  - ✓ **Scenario 1** : One – ambulance decentralization
    - ➢ One of the BSVs was moved to area WB
    - ➢ The other ambulances remained in the Central base
  - ✓ **Scenario 2** : Two- ambulance decentralization
    - ➢ Enlarging the decentralization to two BSVs
    - ➢ Repositioning the two BSVs in WB
  - ✓ **Scenario 3 :** Three- ambulance decentralization
    - ➢ Moving another BSV to area NB
  - ✓ **Other scenario**
    - ➢ Four BSVs were decentralized in areas NB; SB; WB and WB

Evaluation of alternative scenarios



Layer for The BSVs



Layer for The ASVs

Moved and Repositioning the BSVs

Evaluation of alternative scenarios

- Ambulance decentralization

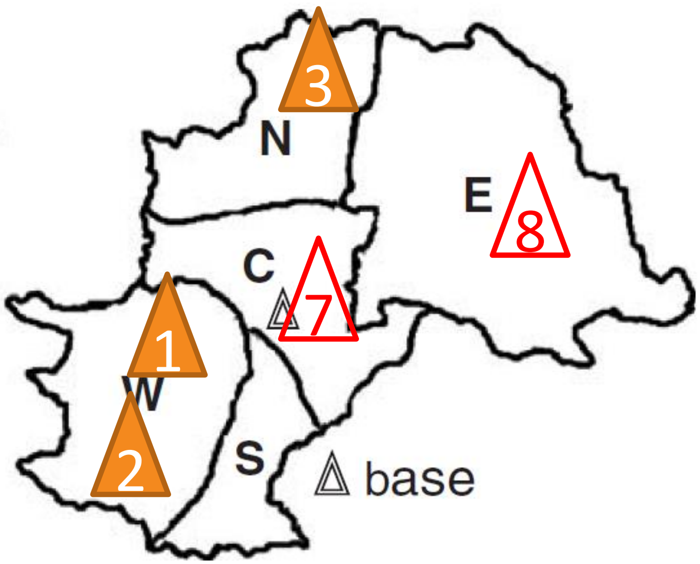
Table 5
Workloads and the mean travel times of ambulances

Ambulance	Scenario 0		Scenario 1		Deviation	
	Workload	Travel time (min)	Workload	Travel time (min)	Minutes	%
1	0.39	10.9	0.39	11.3	0.4	3.7
2	0.39	10.9	0.41	10.6	−0.3	−2.8
3	0.63	14.2	0.64	14.2	0.0	0.0
4	0.63	14.2	0.63	14.2	0.0	0.0
5	0.63	14.2	0.66	8.8	−5.4	−38.0
6	0.63	14.2	0.64	14.2	0.0	0.0
7	0.63	14.2	0.65	14.2	0.0	0.0
8	0.63	14.2	0.65	14.2	0.0	0.0
9	0.63	14.2	0.64	14.2	0.0	0.0
10	0.63	14.2	0.63	14.2	0.0	0.0
Average ASV	0.39	10.9	0.40	11.0	0.1	0.9
Average BSV	0.63	14.2	0.64	13.5	−0.7	−5.0
SYSTEM	0.58	13.6	0.58	13.0	−0.6	−4.4

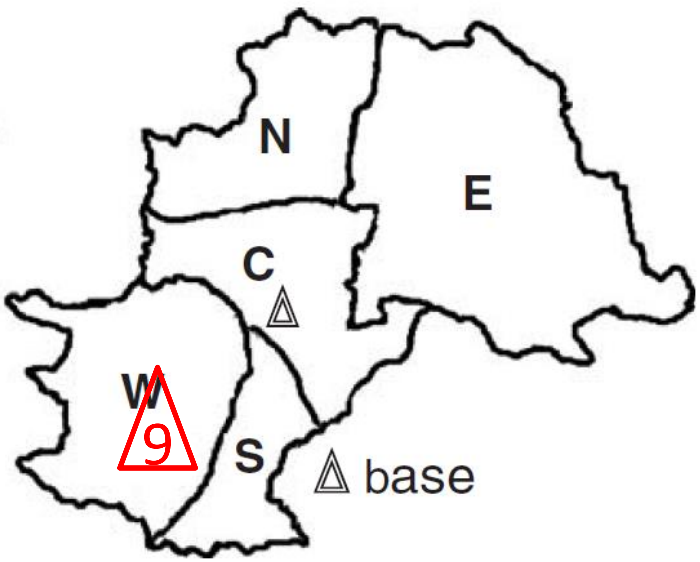
Evaluation of alternative scenarios

- Increasing the number of ambulances
  - ✓ **Scenario 7** : One – ambulance addition
    - ➢ Considering the addition of a new BSV(numbered 11)
    - ➢ Located in Central area
  - ✓ **Scenario 8** : Two- ambulance decentralization
    - ➢ Considering a new BSV(numbered 12) in scenario 7
    - ➢ Located in area EB
  - ✓ **Scenario 9 :** Three- ambulance decentralization
    - ➢ Considering a new ASV(numbered 13) in scenario 8
    - ➢ Located in WA

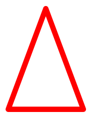
Evaluation of alternative scenarios



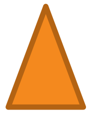
Layer for The BSVs



Layer for The ASVs



The addition of a new BSVs(7,8), ASV(9)



Moved and Repositioning the BSVs

Evaluation of alternative scenarios

Table 6
Mean travel times of ambulances (TU_n)

Ambulance	Travel times (minutes) Scenario									
	0	1	2	3	4	5	6	7	8	9
1	10.9	11.3	11.4	11.3	10.9	10.9	10.9	10.0	7.7	7.5
2	10.9	10.6	10.5	10.6	10.9	10.9	10.8	9.4	7.6	7.1
3	14.2	14.2	10.0	9.0	11.7	11.7	11.6	11.0	8.9	8.8
4	14.2	14.2	13.9	9.7	12.4	12.8	11.3	9.9	9.2	8.4
5	14.2	8.8	9.2	13.8	10.7	10.9	11.4	9.5	9.1	7.7
6	14.2	14.2	14.0	9.7	12.0	10.5	10.5	9.6	8.9	7.9
7	14.2	14.2	14.1	13.9	9.9	12.0	10.4	9.3	8.9	7.8
8	14.2	14.2	14.0	14.0	12.7	12.0	12.7	11.4	8.1	8.1
9	14.2	14.2	13.8	13.8	13.0	10.5	12.6	11.2	8.2	8.2
10	14.2	14.2	13.8	13.9	12.9	12.1	11.6	11.2	8.8	8.7
11								10.5	8.5	8.2
12									8.7	8.2
13										6.6
Average ASV	10.9	11.0	11.0	11.0	10.9	10.9	10.9	9.7	7.7	7.1
Average BSV	14.2	13.5	12.9	12.2	11.9	11.6	11.5	10.4	8.7	8.2
SYSTEM	13.6	13.0	12.5	12.0	11.7	11.5	11.4	10.3	8.6	7.9

Table 7
Mean travel times to atoms ($\overline{T}_j$)

Atom	Travel times (min) Scenario									
	0	1	2	3	4	5	6	7	8	9
1 - *NB*	16.0.	16.2	16.5	16.2	13.0	13.3	11.0	10.0	7.7	7.5
2 - *NA*	13.1	13.2	13.2	15.2	12.7	12.8	12.5	12.1	8.7	8.2
3 - *SB*	15.9	15.9	15.9	15.8	12.4	12.5	13.2	12.2	9.3	9.1
4 - *sa*	13.0	13.1	13.1	12.9	12.6	12.6	12.7	12.3	9.5	8.7
5 - *EB*	15.9	16.1	16.4	16.2	12.9	13.3	13.1	12.3	10.5	8.9
6 - *EA*	13.0	13.1	13.2	13.1	12.7	12.7	12.7	12.3	10.8	9.5
7 - *WB*	15.8	12.5	10.2	9.8	13.3	11.1	11.5	10.8	9.8	8.6
8 - *WA*	12.8	12.4	12.1	12.6	12.5	12.2	12.3	12.1	8.6	8.6
9 - *CB*	5.5	5.8	6.2	6.1	5.9	7.6	9.0	7.1	6.3	6.3
10 - *CA*	4.3	4.5	4.5	4.0	4.3	4.7	4.9	4.1	3.4	3.4
Average ASV	11.3	11.3	11.2	11.2	11.0	11.0	11.0	10.6	8.2	7.7
Average BSV	13.8	13.3	13.0	12.8	11.5	11.5	11.6	10.5	8.7	8.1

Evaluation of alternative scenarios

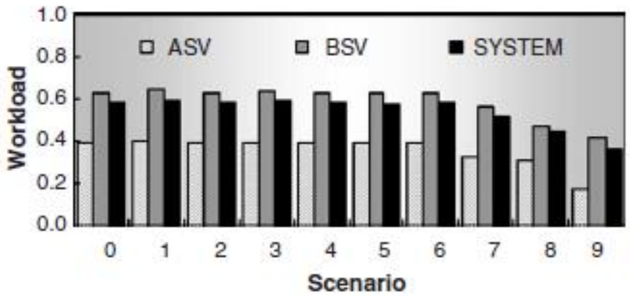


Fig. 4. Ambulance workloads in scenarios 0, 1, . . . , 9.

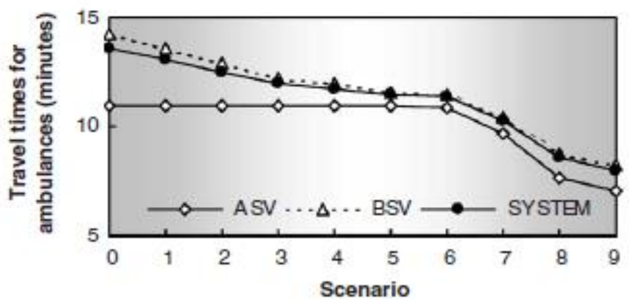


Fig. 5. Mean travel times of ambulances in scenarios 0, 1, . . . , 9.

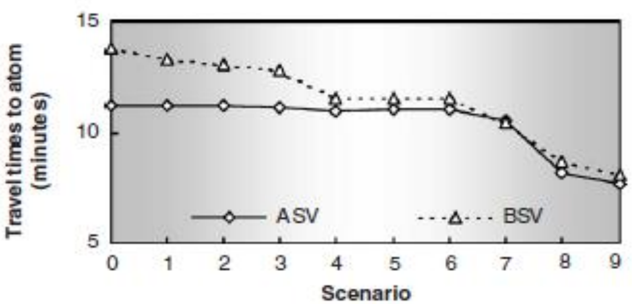


Fig. 6. Mean travel times to atoms in scenarios 0, 1, . . . , 9.

- The decentralization of ambulances increases (from scenario 0–6), the workloads do not change much.
- The workloads reduce substantially by adding new ambulances (scenarios 7–9)

Conclusions

- Summary
  - Studied the application of the hypercube queueing model to the urban EMS of Campinas (SAMU-192) in Brazil
  - To analyze ambulance deployment, in particular, the effects of decentralizing ambulances and increasing their number
- Results
  - Total decentralization as suggested by the system operators of SAMU-192 does not produce satisfactory results.
  - Increasing the number of ambulances in the system leads to significant improvements in system performance measures but obviously at the expense of additional costs and investments.

Thank You