Robust sustainable bi-directional logistics network design under uncertainty

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Contents

Introduction

❖ **Dynamic sustainable bi-directional logistics network design**

Sustainable capacitated facility location problem (SuCFLP)

- A deterministic SuCFLP when the demand and supply are known with certainty.
- Extend the deterministic model to include uncertainty.

→ Robust model formulation (RSuCFLP)

Problem characteristics

- Multiple period are considered
- Capacities can be increase or decreased dynamically over time for all echelons
- Location of facilities and depots can be changed
- Type of depots and their general size (small/medium/large) may be modified
- Forwarded and returned products compete for capacities of facilities and hybrid depots

Introduction

❖ **Reverse logistics network**

SuCFLP and RSuCFLP

- Description of the network structure

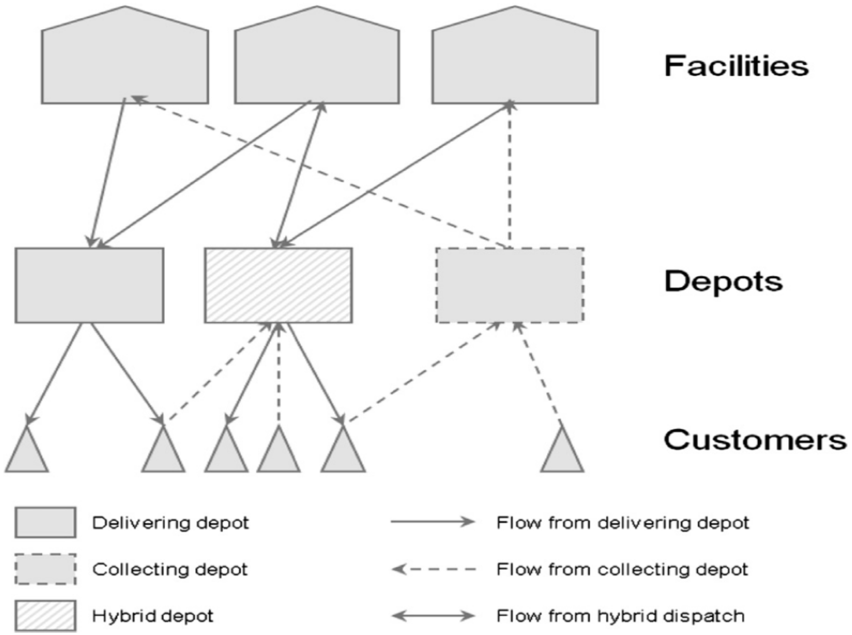

Figure 1. Depiction of bi-directional network structure

Introduction

❖ **Literature review**

Related works

- Sustainable supply chain (design and evaluation sustainable logistics networks: transportation and testing, affecting environmental performance and cost efficiency)

  → Neto et al. (2008), Neto et al. (2010), etc.

- Capacitated facility location problem

  → Aikens (1985), Owen and Daskin (1998), Melo et al. (2004), etc.

- Reverse logistics network design

  → Fleischmann et al. (1997), Lambert et al. (2011), etc.

Recent papers still set their focus exclusively on the design of reverse logistics networks.

→ Integration of reverse into forward-oriented production flows

Problem description

❖ **Deterministic Sustainable capacitated facility location problem (SuCFLP)**

Objective function

- Minimizes costs of an integrated forward and backward production stream.

  → Depot type and size change as well as for opening and closing sites

Decision variables

- Quantity of product
- Capacity adjustments and resulting capacities
- Building and closing a site
- Changes of status (type or size)

Problem description

❖ **Deterministic SuCFLP**

Main constraints

- Capacity constraints (facilities and depots)

  ◄- - - - Min/max, increase/decrease

Assumptions

- Single product type.
- Facilities are assumed to be operating on both forwarded and collected products at the same time.

Problem description

Deterministic SuCFLP

Notation [1]

The following notations are used in the model formulation.
Occurring indices:

I Set of facilities (open and close) ($I{:=}I^o \cup I^c$)
J Set of depots (open and close) ($J{:=}J^o \cup J^c$)
L Set of customers (L)
A Set of depots types ($A{:=}(for, hyb, ret)$)
M Set of depots sizes ($M{:=}(s, m, l)$)
T Planning horizon ($T{:=}\{1, 2, \ldots, \tau\}$)

Parameters of the model:

a_f^f Facility utilization by forwarded product (Euro/Unit)
a_r^{*f} Facility utilization (handling) by collected product (Euro/Unit)
a_f^d Depot utilization by forwarded product (Euro/Unit)
a_r^{*d} Depot utilization (handling) by collected product (Euro/Unit)
b_t Total budget for facility changes in period t
$c_{\alpha\beta}$ Unit shipping costs between location α and β ($c_{ij}^{fd}, c_{ji}^{*df}, c_{jl}^{dc}, c_{lj}^{*cd}$) (Euro/Unit)
d_{lt} Product demand of customer l in period t (units)
s_{lt} Product supply (products to be collected) from customer l in period t (units)
e_i^f Costs to change capacity of facility i per additional unit (Euro/unit)
e_j^d Costs to change capacity of depot j per additional unit (Euro/unit)

f_i^f Fixed costs to open a facility i (Euro)
ξ_i^f Fixed costs to close a facility i (Euro)
f_{jam}^d Fixed costs to open a depot j depending on depot type a and size m (Euro)
ξ_{jam}^d Fixed costs to close a depot j depending on depot type a and size m (Euro)
n_a Fixed costs to change type a of depot (Euro)
k_m Fixed costs to change magnitude m of depot (Euro)
g_i^f Variable maintenance costs of facility i depending on installed capacity (Euro/unit)
g_j^d Variable maintenance costs of depot j depending on installed capacity (Euro/unit)
h_i^f Variable operation costs of facility i depending on processed products (Euro/unit)
h_j^d Variable operation costs of depot j depending on processed products (Euro/unit)
$cap0_i^f$ Starting capacity of facility i (units)
$cmin_i^f$ Minimum capacity of facility i (units)
$cmax_i^f$ Maximum capacity of facility i (units)
$cap0_j^d$ Starting capacity of already open depot j (units)
$cmin_j$ Minimum capacity of depot j (units)
$mcin_m$ Starting capacity of newly opened type m depot (units)
mcx_m Maximum capacity of type m depot (units)

Problem description

❖ Deterministic SuCFLP

Notation [2]

Decision variables:

The decisions consist of mainly four categories. Indicated by the variables x are the decisions on the quantity of products being shipped from or received by a specific site. The variables v and w define the proposed capacity adjustments and resulting capacities of a site, respectively. The decision on building or closing a site is mapped with the variables y^f and y^d. Changes of status, type or size are tracked with the variables z^f, z^d, η and κ. The decision variables of the model are as follows:

$x_{\alpha\beta t}$	Quantity of product shipped in period t from location α to β ($x^{fd}_{ijt}, x^{dc}_{jlt}, x^{df}_{jit}, x^{cd}_{ljt}$) (units)
v^f_{it}	Capacity change in facility i at start of period t (no dimension)
w^f_{it}	Capacity of facility i at the start of period t (units)
v^d_{jt}	Capacity change in depot j at start of period t (no dimension)
w^d_{jt}	Capacity of depot j at the start of period t (units)
y^f_{it}	$\in\{0,1\}$ indicates if facility i is open $\{=1\}$ in period t (no dimension)
y^d_{jatm}	$\in\{0,1\}$ indicates type, size and if depot j is open $\{=1\}$ in period t (no dimension)
z^d_{jatm}	$\in\{0,1\}$ tracks status changes $\{=1\}$ for depot j from period $t-1$ to t (no dimension)
z^f_{it}	$\in\{0,1\}$ tracks status changes $\{=1\}$ for facility i from period $t-1$ to t (no dimension)
η_{jat}	$\in\{0,1\}$ tracks type changes $\{=1\}$ for depot j from period $t-1$ to t (no dimension)
κ_{jtm}	$\in\{0,1\}$ tracks magnitude changes $\{=1\}$ for depot j from period $t-1$ to t (no dimension)

Problem description

- **Mathematical model**

SuCFLP [1]

The objective function is formulated in the following equation:

$$
Min\left\{
\begin{array}{l}
\sum\limits_{\substack{i\in I, j\in J,\\ t\in T}} (c_{ij}^{fd} x_{ijt}^{fd} + c_{ji}^{*df} x_{jit}^{df}) + \sum\limits_{\substack{j\in J, l\in L,\\ t\in T}} (c_{jl}^{dc} x_{jlt}^{dc} + c_{lj}^{*cd} x_{ljt}^{cd}) \\
+ \sum\limits_{i\in I^c, t\in T} f_i^f z_{it}^f + \sum\limits_{i\in I^o, t\in T} \xi_i^f z_{it}^f \\
+ \sum\limits_{\substack{j\in J^c, t\in T,\\ a\in A, m\in M}} f_{jam}^d z_{jatm}^d + \sum\limits_{\substack{j\in J^o, t\in T,\\ a\in A, m\in M}} \xi_{jam}^d z_{jatm}^d + \sum\limits_{\substack{j\in J, m\in M,\\ t\in T}} k_m \kappa_{jtm} \\
+ \sum\limits_{\substack{j\in J, a\in A,\\ t\in T}} n_a \eta_{jat} + \sum\limits_{i\in I, t\in T} e_i^f v_{it}^f + \sum\limits_{j\in J, t\in T} e_j^d v_{jt}^d \\
+ \sum\limits_{i\in I, t\in T} g_i^f w_{it}^f + \sum\limits_{j\in J, t\in T} g_j^d w_{jt}^d + \sum\limits_{i\in I, t\in T} h_i^f \sum\limits_{j\in J} (x_{ijt}^{fd} + x_{jit}^{df}) \\
+ \sum\limits_{j\in J, t\in T} h_j^d \sum\limits_{l\in L} (x_{jlt}^{dc} + x_{ljt}^{cd})
\end{array}
\right\} \quad (1)
$$

Subject to

$$\sum_{j\in J} x_{jlt}^{dc} = d_{lt} \quad \forall l\in L, t\in T \quad (2)$$

$$\sum_{j\in J} x_{ljt}^{cd} = s_{lt} \quad \forall l\in L, t\in T \quad (3)$$

$$\sum_{i\in I} x_{ijt}^{fd} = \sum_{l\in L} x_{jlt}^{dc} \quad \forall j\in J, t\in T \quad (4)$$

$$\sum_{i\in I} x_{jit}^{df} = \sum_{l\in L} x_{ljt}^{cd} \quad \forall j\in J, t\in T \quad (5)$$

$$\sum_{j\in J} (a_f^f x_{ijt}^{fd} + a_r^{*f} x_{jit}^{df}) \le y_{it}^f \, cmax_i^f \quad \forall i\in I, t\in T \quad (6)$$

$$\sum_{i\in I} a_f^d x_{ijt}^{fd} \le \sum_{\substack{a\in A\setminus\{ret\},\\ m\in M}} y_{jatm}^d mcx_m \quad \forall j\in J, t\in T \quad (7)$$

$$\sum_{l\in L} a_f^d x_{jlt}^{dc} \le \sum_{\substack{a\in A\setminus\{ret\},\\ m\in M}} y_{jatm}^d mcx_m \quad \forall j\in J, t\in T \quad (8)$$

Problem description

- **Mathematical model**

SuCFLP [2]

$$\sum_{i \in I} a_f^d x_{ijt}^{fd} \leq \sum_{m \in M} (1 - y_{j,\mathbf{ret},tm}^d) mcx_m \quad \forall j \in J, t \in T \tag{9}$$

$$\sum_{l \in L} a_f^d x_{jlt}^{dc} \leq \sum_{m \in M} (1 - y_{j,\mathbf{ret},tm}^d) mcx_m \quad \forall j \in J, t \in T \tag{10}$$

$$\sum_{i \in I} a_r^{*d} x_{jit}^{df} \leq \sum_{\substack{a \in A \setminus \{for\}, \\ m \in M}} y_{jatm}^d mcx_m \quad \forall j \in J, t \in T \tag{11}$$

$$\sum_{l \in L} a_r^{*d} x_{ljt}^{cd} \leq \sum_{\substack{a \in A \setminus \{for\}, \\ m \in M}} y_{jatm}^d mcx_m \quad \forall j \in J, t \in T \tag{12}$$

$$\sum_{i \in I} a_r^{*d} x_{jit}^{df} \leq \sum_{m \in M} (1 - y_{j,\mathbf{for},tm}^d) mcx_m \quad \forall j \in J, t \in T \tag{13}$$

$$\sum_{l \in L} a_r^{*d} x_{ljt}^{cd} \leq \sum_{m \in M} (1 - y_{j,\mathbf{for},tm}^d) mcx_m \quad \forall j \in J, t \in T \tag{14}$$

$$z_{it}^f = y_{i,t-1}^f - y_{it}^f \quad \forall i \in I^o, t \in T \tag{15}$$

$$z_{it}^f = y_{it}^f - y_{i,t-1}^f \quad \forall i \in I^c, t \in T \tag{16}$$

$$z_{jatm}^d \geq y_{jatm}^d - \sum_{\substack{b \in A, \\ m \in M}} y_{jb,t-1,m}^d \quad \forall j \in J^c, a \in A, m \in M, t \in T \tag{17}$$

$$z_{jatm}^d \leq \sum_{b \in A} (y_{jb,t-1,m}^d - y_{jbtm}^d) \quad \forall j \in J^o, a \in A, m \in M, t \in T \tag{18}$$

$$z_{jatm}^d \geq y_{ja,t-1,m}^d - \sum_{b \in A, m \in M} (y_{jbtm}^d) \quad \forall j \in J^o, a \in A, m \in M, t \in T \tag{19}$$

$$\eta_{jat} \geq y_{jatm}^d - y_{ja,t-1,m}^d \quad \forall j \in J^o, a \in A, m \in M, t \in T \tag{20}$$

$$\eta_{jat} \geq y_{jatm}^d - y_{ja,t-1,m}^d - z_{jatm}^d \quad \forall j \in J^c, a \in A, m \in M, t \in T \tag{21}$$

$$\kappa_{jtm} \geq y_{jatm}^d - y_{ja,t-1,m}^d - z_{jatm}^d \quad \forall j \in J^c, a \in A, m \in M, t \in T \tag{22}$$

$$\kappa_{jtm} \geq y_{jatm}^d - y_{ja,t-1,m}^d \quad \forall j \in J^o, a \in A, m \in M, t \in T \tag{23}$$

$$\sum_{a \in A, m \in M} y_{jatm}^d \leq 1 \quad \forall j \in J, t \in T \tag{24}$$

$$w_{it}^f \leq cap0_i^f + \sum_{r \in T, r \leq t} v_{ir}^f \quad \forall i \in I, t \in T \tag{25}$$

$$w_{it}^f \leq y_{it}^f cmax_i^f \quad \forall i \in I, t \in T \tag{26}$$

$$w_{it}^f \geq y_{it}^f cmin_i^f \quad \forall i \in I, t \in T \tag{27}$$

$$w_{jt}^d \leq \sum_{a \in A, m \in M} y_{jatm}^d mcx_m \quad \forall j \in J, t \in T \tag{28}$$

$$w_{jt}^d \geq \sum_{a \in A, m \in M} y_{jatm}^d cmin_j \quad \forall j \in J, t \in T \tag{29}$$

$$w_{jt}^d \leq cap0_j^d + \sum_{\substack{k \in T, \\ k \leq t}} v_{jk}^d + \sum_{\substack{a \in A, \\ m \in M}} z_{jatm}^d mcin_m \quad \forall j \in J, t \in T \tag{30}$$

Problem description

❖ **Mathematical model**

SuCFLP [3]

$$\sum_{j\in J}(a_f^f x_{ijt}^{fd} + a_r^{*f} x_{jit}^{df}) \le w_{it}^f \quad \forall i\in I, t\in T \tag{31}$$

$$\sum_{l\in L}(a_f^d x_{jlt}^{dc} + a_r^{*d} x_{ljt}^{cd}) \le w_{jt}^d \quad \forall j\in J, t\in T \tag{32}$$

$$\begin{aligned}\Bigg(\bigg(\sum_{i\in I^c} f_i^f + \sum_{i\in I^o} \xi_i^f\bigg) z_{it}^f + \bigg(\sum_{\substack{j\in J^c, a\in A,\\ m\in M}} f_{jam}^d + \sum_{\substack{j\in J^o, a\in A,\\ m\in M}} \xi_{jam}^d\bigg) z_{jatm}^d \\ + \sum_{i\in I} e_i^f v_{it}^f + \sum_{j\in J} e_j^d v_{jt}^d + \sum_{\substack{j\in J,\\ m\in M}} k_m \kappa_{jtm} + \sum_{\substack{j\in J,\\ a\in A}} n_a \eta_{jat}\Bigg) \le b_t \quad \forall t\in T\end{aligned} \tag{33}$$

$$\begin{aligned}x_{ijt}^{fd}, x_{jit}^{df}, x_{jlt}^{dc}, x_{ljt}^{cd}, v_{it}^f, v_{jt}^d, w_{it}^f, w_{jt}^d, z_{it}^f y_{it}^f, y_{jatm}^d, z_{jatm}^d, \eta_{jat}, mgch_{jtm} \ge 0 \\ \forall i\in I, j\in J, a\in A, m\in M, t\in T\end{aligned} \tag{34}$$

Problem description

❖ **Robust sustainable capacitated facility location problem (RSuCFLP)**

Notation [1]

Additional index:

S Set of possible demand and supply scenarios

Additional parameters of the robust model:

d_{lts} Product demand of customer l in period t and scenario s (units)

sup_{lts} Product supply from customer l in period t and scenario s (units)

p_s Probability of specific scenario to occur, $\sum_{s\in S} p_s = 1$

$\hat{W}_s$ Best objective value that can be obtained under scenario s (Euro)

λ_i Costs per unit shortage of capacity in facility i (Euro)

The additional decision variables of the robust model are as follows:

u^f_{its} Shortage of capacity in facility i in period t in scenario s. With $u \geq 0$ (units)

Re_s Regret associated with scenario s and the current solution

Problem description

- **Mathematical model**

RSuCFLP [1]

$$\text{Min}\left\{\sum_{s\in S} p_s \frac{Re_s}{\hat{W}_s}\right\} \tag{35}$$

Subject to (15)–(24) and

$$Re_s = \left\{\begin{array}{l} \sum_{\substack{i\in I, j\in J,\\ t\in T}} (c_{ij}^{fd} x_{ijts}^{fd} + c_{ji}^{*df} x_{jits}^{df}) + \sum_{\substack{j\in J, l\in L,\\ t\in T}} (c_{jl}^{dc} x_{jlts}^{dc} + c_{lj}^{*cd} x_{ljts}^{cd}) \\ + \sum_{i\in I^c, t\in T} f_i^f z_{it}^f + \sum_{i\in I^o, t\in T} \xi_i^f z_{it}^f \\ + \sum_{\substack{j\in J^c, t\in T,\\ a\in A, m\in M}} f_{jam}^d z_{jatm}^d + \sum_{\substack{j\in J^o, t\in T,\\ a\in A, m\in M}} \xi_{jam}^d z_{jatm}^d + \sum_{\substack{j\in J, m\in M,\\ t\in T}} k_m \kappa_{jtm} \\ + \sum_{\substack{j\in J, a\in A,\\ t\in T}} n_a \eta_{jat} + \sum_{j\in J, t\in T} e_j^d v_{jts}^d + \sum_{i\in I, t\in T} e_i^f v_{its}^f \\ + \sum_{i\in I, t\in T} g_i^f w_{its}^f + \sum_{j\in J, t\in T} g_j^d w_{jts}^d \\ + \sum_{i\in I, t\in T} h_i^f \sum_{j\in J} (x_{ijts}^{fd} + x_{jits}^{df}) + \sum_{j\in J, t\in T} h_j^d \sum_{l\in L} (x_{jlts}^{dc} + x_{ljts}^{cd}) \\ + \sum_{i\in I, t\in T} \lambda_i u_{its}^f - \hat{W}_s \end{array}\right\} \forall s\in S \tag{36}$$

$$\sum_{j\in J} (a_f^f x_{ijts}^{fd} + a_r^{*f} x_{jits}^{df}) - u_{its}^f \le w_{its}^f \quad \forall i\in I, t\in T, s\in S \tag{37}$$

$$u_{its}^f \le y_{it}^f \, cmax_i^f \quad \forall i\in I, t\in T, s\in S \tag{38}$$

Model application and results

Experiments and results

Example

Performances of the robust and deterministic model are compared.

→ Six different and varying scenarios of future business developments are implemented.

✓ Solution method

CPLEX

Results

Table 1. Expected cost rankings .

Scenario Configuration	Scenario 1 Gap ratio (rank)	Scenario 2 Gap ratio (rank)	Scenario 3 Gap ratio (rank)	Scenario 4 Gap ratio (rank)	Scenario 5 Gap ratio (rank)	Scenario 6 Gap ratio (rank)	Expected costs Gap ratio (rank)
D1	Opt. sol.	0.05 (1)	2.06 (4)	2.09 (4)	5.63 (6)	4.97 (6)	1.09 (4)
D2	0.04 (2)	Opt. sol.	0.71 (3)	0.84 (2)	2.22 (5)	2.06 (5)	0.42 (2)
D3	10.22 (4)	2.23 (3)	Opt. sol.	1.27 (3)	0.22 (1)	0.42 (2)	3.68 (5)
D4	1.09 (3)	2.24 (4)	0.31 (2)	Opt. sol.	0.63 (4)	0.83 (4)	0.91 (3)
D5	25.92 (5)	17.14 (5)	4.17 (5)	16.56 (5)	Opt. sol.	0.03 (1)	16.62 (6)
D6	29.59 (6)	20.65 (6)	6.1 (6)	19.75 (6)	0.24 (2)	Opt. sol.	19.73 (7)
R	0 (1)	0.05 (2)	0.18 (1)	0.04 (1)	0.45 (3)	0.65 (3)	0.07(1)

Conclusion

❖ **Summary**

- Introduced two Capacitated Facility Location Problem models as a mean to enable companies to assess the implementation of product recovery and disposal into the existing supply chain.

- The computational tests indicate that depending on the test configuration and declaration of design variables, differences between the deterministic and robust model solutions might be small.

Thank You !

Q and A