# Location-routing: Issues, models and methods

European Journal of Operational Research, 2007, 177, 649-672.

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**August, 21, 2012**

# Contents

# Introduction

❖ **What is location-routing problem?**

Definition

- Set of problems
- Approach to modelling and solving locational problems

◄---- not a single well defined problem

e.g., Weber problem or the travelling salesman problem

Hierarchical viewpoint

- To solve a facility location problem (the ‘‘master problem’’)
- Simultaneously need to solve a vehicle routing problem (the ‘‘subproblem’’)

→ Location, routing, location-routing

Set of problems

- Practical viewpoint: distribution management
- Mathematical viewpoint: combinatorial optimisation problem
- NP-hard problem: it encompasses two NP-hard problems

# Introduction

- **Variety of location-routing problem versions**

ILP formulations for various LRP problems

| Problem type | Paper | Section |
|---|---|---|
| Stochastic LRP | Laporte et al. (1989) | 5.2 |
| Dynamic LRP | Laporte and Dejax (1989) | 5.3 |
| Hamiltonian *p*-median | Branco and Coelho (1990) | 4.2 |
| Road-train routing | Semet (1995) | 6.4 |
| Vehicle routing-allocation (VRAP) | Beasley and Nascimento (1996) | 6.3 |
| Many-to-many LRP | Nagy and Salhi (1998) | 6.2 |
| Eulerian location | Ghiani and Laporte (1999) | 3 |
| LRP with mixed fleet | Wu et al. (2002) | 4.3 |
| Location-routing-inventory | Liu and Lee (2003) | 5.2 |
| Plant cycle location | Labbé et al. (2004) | 3 |
| Many-to-many LRP | Wasner and Zäpfel (2004) | 6.2 |
| Multi-level location-routing-inventory | Ambrosino and Scutellà (2005) | 6.4 |
| Deterministic LRP | Albareda-Sambola et al. (2005) | 4.4 |
| VRAP (median cycle problem) | Labbé et al. (2005) | 6.3 |
| LRP with non-linear costs | Melechovský et al. (2005) | 4.4 |
| Planar LRP (single-depot) | Schwardt and Dethloff (2005) | 4.2 |
| Restricted VRAP | Gunnarsson et al. (in press) | 6.3 |
| Planar LRP (multi-depot) | Salhi and Nagy (in review) | 4.3 |

# Introduction

## ❖ Applications of location-routing

A summary of LRP applications

| Paper | Section | Application area | Country/region | Facilities | Customers |
|---|---|---|---|---|---|
| Watson-Gandy and Dohrn (1973) | 4.4 | Food and drink distribution | United Kingdom | 40 | 300 |
| Bednar and Strohmeier (1979) | 4.2 | Consumer goods distribution | Austria | 3 | 50 |
| Or and Pierskalla (1979) | 4.4 | Blood bank location | United States | 3 | 117 |
| Jacobsen and Madsen (1980) | 6.4 | Newspaper distribution | Denmark | 42 | 4510 |
| Nambiar et al. (1981) | 4.1/4.4 | Rubber plant location | Malaysia | 15 | 300 |
| Perl and Daskin (1984, 1985) | 4.3 | Goods distribution | United States | 4 | 318 |
| Labbé and Laporte (1986) | 6.3 | Postbox location | Belgium | Not given | Not given |
| Nambiar et al. (1989) | 5.3 | Rubber plant location | Malaysia | 10 | 47 |
| Semet and Taillard (1993) | 6.4 | Grocery distribution | Switzerland | 9 | 90 |
| Kulcar (1996) | 6.1 | Waste collection | Belgium | 13 | 260 |
| Murty and Djang (1999) | 6.3 | Military equipment location | United States | 29 | 331 |
| Bruns et al. (2000) | 6.2 | Parcel delivery | Switzerland | 200 | 3200 |
| Chan et al. (2001) | 5.3 | Medical evacuation | United States | 9 | 52 |
| Lin et al. (2002) | 4.4 | Bill delivery | Hong Kong | 4 | 27 |
| Lee et al. (2003) | 3 | Optical network design | Korea | 50 | 50 |
| Wasner and Zäpfel (2004) | 6.2 | Parcel delivery | Austria | 10 | 2042 |
| Billionnet et al. (2005) | 3 | Telecom network design | France | 6 | 70 |
| Gunnarsson et al. (in press) | 6.3 | Shipping industry | Europe | 24 | 300 |
| Lischak and Triesch (in review) | 6.2 | Parcel delivery | Poland | 22 | 750 |

# Model classifications

## ❖ Classification

Classification criteria

- Problem structure
- Pertaining to the location of facilities
- Patern of vehicle routes or to the entirety of the problem

◄---- Four key aspects of location-routing problem

✓ Hierarchical structure

Facilities servicing a number of customers, these are connected to their depot by means of vehicle tours.

→ Single or multi level, etc.

✓ Type of input data

This may be deterministic or stochastic.

→ All stochastic papers consider customer demand as the only stochastic variable

# Model classifications

❖ **Classification**

Classification criteria (continued)

✓ Planning period

This may be single-period or multi-period.

→ Single or multiple periods are known respectively as static or dynamic

✓ Solution method

This may be exact or heuristic.

→ More papers using heuristic methods, but exact methods are often very successful for special cases of the LRP

✓ Objective function

Usual objective for location-routing problems is that of overall cost minimization.

→ Costs can be divided into depot costs and vehicle costs

# Model classifications

❖ **Classification**

Classification criteria (continued)

✓ Solution space

This can be discrete, network or continuous.

→ Most of the LRP literature deals with discrete location

✓ Number of depots

This may be single or multiple.

→ Most papers on the LRP deal with multiple depots

✓ Number and types of vehicles

Most location-routing problems, the number of vehicles is not fixed in advance and a homogeneous fleet is assumed.

→ Homogeneous or heterogeneous, fixed number of vehicle etc.

# Model classifications

- **Classification**

  Classification criteria (continued)

  - ✓ Route structure

    To start out from a depot, traverse through a number of customer nodes, delivering goods at each customer and finally return to the same depot.

    → e.g., arc routing, multiple trips, delivery and pickup, etc (exceptions)

# Various LRP problems

❖ **Various LRP problems with classification criteria**

Eulerian location problem

- Instead of consisting of customer nodes, require the vehicles to traverse given edges.
- Vehicles are required to traverse given edges (rather than nodes).
- Depots are located based on the partition network and an attractiveness measure relating to the number and weight of the arcs incident to potential depot nodes.

◄---- Underlying routing problem is the arc routing problem

e.g., Levy and Bodin (1989) – clustering based methods in heuristic solution method for deterministic problems

Plant-cycle location problem

- Which plants to open, the assignment of customers to plants, and the cycles containing each open plant and its customers
- Minimizing the total cost

◄---- Related to the well-known Capacitated Facility Location Problem

e.g., Billionnet et al. (2005) – exact method for deterministic problems

# Various LRP problems

❖ **Various LRP problems with classification criteria**

Hamiltonian p-median problem

- Where exactly p facilities must be located and each facility has exactly one route Minimizing the total cost.
- A special case of the LRP

◄---- Partitioning the customer set into p Hamiltonian circuits

e.g., Branco and Coelho (1990) – clustering based methods in heuristic solution method for deterministic problems

Planar LRP (single or multi-depot)

- Location of facilities in the plane and the determine the vehicle routes originating from the chosen facility to serve all customers
- Minimum total transportation cost

◄---- Concerned with locating the facilities in the continuous space

e.g., Schwardt and Dethloff (2005) – clustering based methods in heuristic solution method for deterministic problems

# Various LRP problems

❖ **Various LRP problems with classification criteria**

Many-to-many location-routing problem

- Several customers wish to send goods to others.
- A network of hubs is to be located.
- Each customer sends a different commodity to every other customer.

◄---- Generalized version of hub location problem (parcel delivery, delivery and pickup)

e.g., Nagy and Salhi (1998) – Problems with non-standard hierarchical structure

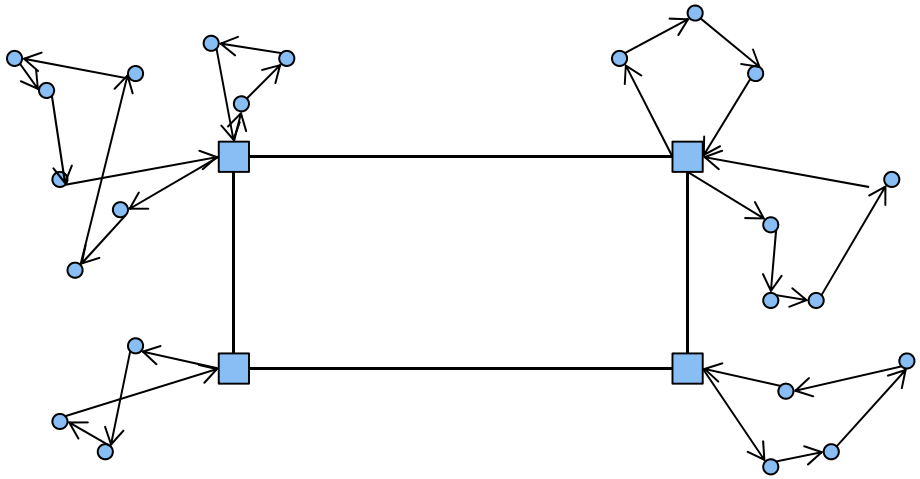

Figure 1. An illustration of the many-to-many location-routing problem

# Various LRP problems

❖ **Various LRP problems with classification criteria**

Vehicle routing-allocation problems

- VRAP not all customers need be visited by the vehicles.
- However customers not visited either have to be allocated to some customer on one of the vehicle tours or left isolated.

◄---- Median cycle problem (bus lines in rural or urban areas)

e.g., Beasley and Nascimento (1996) – Problems with non-standard hierarchical structure

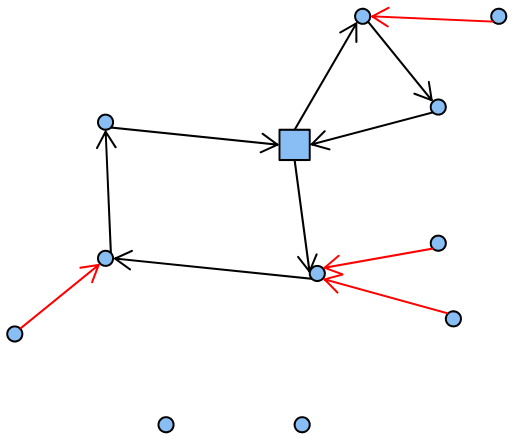

Figure 2. An illustration of the vehicle routing-allocation problem

# Various LRP problems

- **Various LRP problems with classification criteria**

Multi-level location-routing problems

- Routing occurs both at hub and customer level.
- Two-level location-routing problem.
- Newspapers are delivered from the factory to transfer points and from these to the customers.

◄---- Road-train routing problem

e.g., Jacobsen and Madsen (1980) and Madsen(1983) – Problems with non-standard hierarchical structure

# Conclusion

- **Summary**
  - Survey of location-routing
  - Propose a classification scheme
  - A number of problem variants
  - Some suggestions for future research

# Thank You !

*Q and A*