Multi-period reverse logistics network design

European Journal of Operational Research, 2012, 220, 67-78.

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August, 01, 2012

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❖ Scope of research

- A reverse logistics network

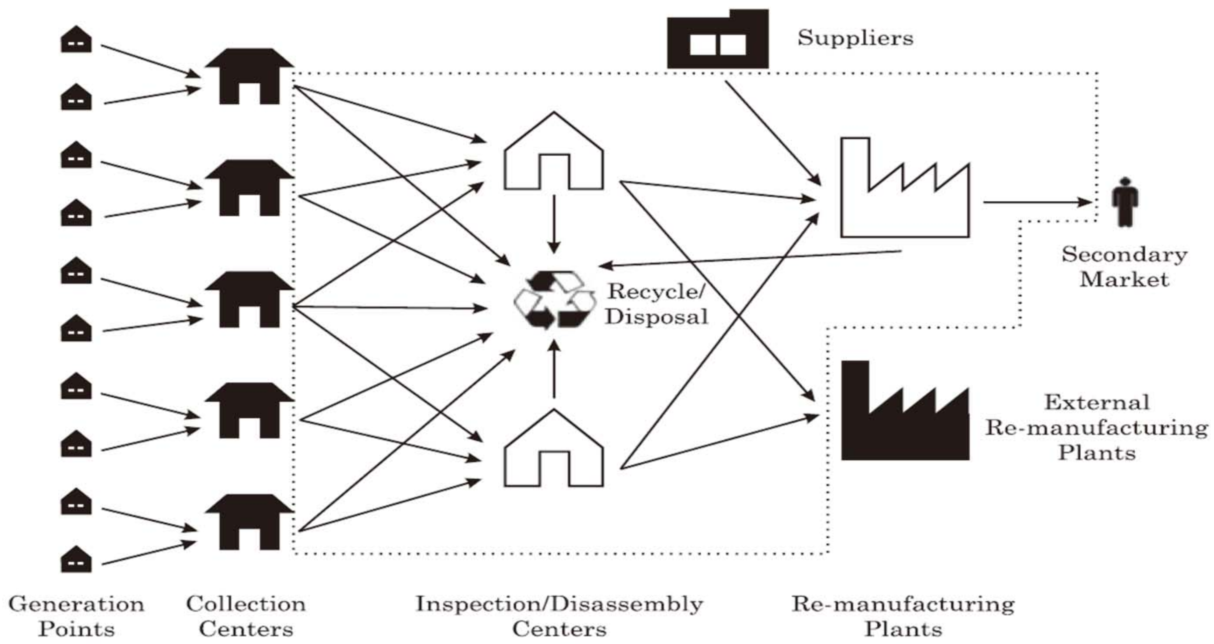

Figure 1. A reverse logistics network

Introduction

- Scope of research (continued)
  - Multi-period reverse logistics network design
    - Determination of the optimal site and capacities of facilities/plants.
    - Amount of returns to be disposed, recycled, and remanufactured by third parties are determined simultaneously.
    - Allow gradual changes in the network structure (i.e., capacity expansions).
    - Multi-period setting and multi-commodity (with reverse bill of materials).

Introduction

❖ Literature review

Table 1. Related articles

Reverse logistics features in addition to the location of the reverse facilities.

Article	Location decisions for reverse activities			Multiple products	Reverse BOM	Dynamic returns	Dynamic location	Capacities	Time adjustment capacities	Minimum throughput	Profit oriented	Secondary market
	Inspection disassembly	Recycling	Remanufacturing refurbishing									
Jayaraman et al. (1999)			√	√				C				
Krikke et al. (1999)			√									
Louwers et al. (1999)		√		√								√
Fleischmann et al. (2001)	√		√									
Schultmann et al. (2003)	√				√			C				
Krikke et al. (2003)	√	√	√	√	√							
Realff et al. (2004)	√	√	√	√	√	√		C			√	√
Listeş and Dekker (2005)	√	√		√				C			√	√
Lieckens and Vandaele (2007)		√	√					ML			√	√
Figueiredo and Mayerle (2008)	√											
Pati et al. (2008)	√	√		√				C				
Salema et al. (2009)	√		√	√		√		C		√	√	
Srivastava (2008)			√	√				C	√		√	
Fonseca et al. (2010)	√	√		√	√			M				
Gomes et al. (2011)	√	√		√	√	√		C				
The new model	√	√	√	√	√	√	√	M	√	√	√	√

'C': Capacitated
'M': Modular capacities
'ML': Multi-level capacities

Problem description

❖ Multi-period reverse logistics network design

- Various features of practical relevance
  - A multi-period setting, modular capacities, capacity expansion of facilities, reverse bill of materials, minimum throughput at the facilities, variable operational costs, finite demand in the secondary market, and a profit oriented objective function.

- Objective function
  - Maximizing profit (revenues – costs)
    - Revenues: from the recycling centers, from the external remanufacturing plants, from the secondary market.
    - Costs: fixed costs of establishing facilities and capacity modules, operational costs, transportation costs, inventory holding costs, and component purchasing costs.

Problem description

❖ Multi-period reverse logistics network design (continued)

- Decision variables
  - When, where, and how many facilities to locate with which capacities.
  - When to invest for the capacity expansion for the facilities.
  - Amounts of products or components to send to recycling facilities and external remanufacturing plants.
  - Amount of components to purchase for the remanufacturing plants.
- Main constraints
  - Capacities of facilities, minimum throughput, and inventory holding capacity.

Problem description

❖ Notation

Revenues

PRG_p^t	unit revenue from product $p \in P$ recycled from a collection center in period $t \in T$
PRI_c^t	unit revenue from component $c \in C$ recycled from an inspection center in period $t \in T$
PRR_c^t	unit revenue from component $c \in C$ recycled from a remanufacturing plant in period $t \in T$
PER_{ip}^t	unit revenue from product $p \in P$ sold to an external remanufacturing plant at $i \in ER$ in period $t \in T$
PSM_p^t	unit revenue from product $p \in P$ sold to the secondary market in period $t \in T$

Costs

FI_i^t	set-up cost for installing an inspection center at $i \in I^I$ in the beginning of period $t \in T$
FR_i^t	set-up cost for installing a remanufacturing plant at $i \in I^R$ in the beginning of period $t \in T$
FKI_{iq}^t	set-up cost for a module of type $q \in Q^I$ to be added to an inspection center located at $i \in I^I$ in period $t \in T$
FKR_{iq}^t	set-up cost for a module of type $q \in Q^R$ to be added to a remanufacturing plant located at $i \in I^R$ in period $t \in T$
OI_{ip}^t	cost for operating one unit of product $p \in P$ in an inspection center $i \in I^I$ in period $t \in T$
OR_{ip}^t	cost for producing one unit of product $p \in P$ in a remanufacturing plant $i \in I^R$ in period $t \in T$
T_{ijp}^t	unit transportation cost of product $p \in P$ (component $p \in C$) from $i \in I^G$ to $j \in I^I$, or $i \in I^I$ to $j \in I^R$ in period $t \in T$
IC_{ic}^t	unit inventory holding cost for component $c \in C$ in a remenufacturing plant $i \in I^R$ in period $t \in T$
BC_{ic}^t	cost of purchasing one unit of component $c \in C$ for remanufacturing plant $i \in I^R$ in period $t \in T$

Decision variables

x_{ijp}^t	amount of product $p \in P$ (component $p \in C$) shipped from site i to site j, $(i,j) \in A$, in period $t \in T$
I_{ic}^t	amount of component $c \in C$ hold in inventory in remanufacturing plant $i \in I^R$ in the end of period $t \in T$
b_{ic}^t	amount of component $c \in C$ purchased for remanufacturing plant $i \in I^R$ in the beginning of period $t \in T$

$$y_i^t = \begin{cases} 1 & \text{If an inspection center } i \in I^I \text{ is operating in period } t \in T, \\ 0 & \text{otherwise.} \end{cases}$$

$$z_i^t = \begin{cases} 1 & \text{If a remanufacturing plant } i \in I^R \text{ is operating in period } t \in T, \\ 0 & \text{otherwise.} \end{cases}$$

$$u_{iq}^t = \begin{cases} 1 & \text{If a module of type } q \in Q^I \text{ is added to an inspection center } i \in I^I, \text{ in the beginning of period } t \in T, \\ 0 & \text{otherwise.} \end{cases}$$

$$v_{iq}^t = \begin{cases} 1 & \text{If a module of type } q \in Q^R \text{ is added to a remanufacturing center } i \in I^R, \text{ in the beginning of period } t \in T, \\ 0 & \text{otherwise.} \end{cases}$$

Problem description

❖ Mathematical formulation

$$
\begin{aligned}
Max \sum_{t\in T}\Bigg[&\sum_{p\in P}\sum_{i\in I^G} PRG_p^t x^t_{iR^Gp} + \sum_{c\in C}\sum_{i\in I^I} PRI_c^t x^t_{iR^Ic} + \sum_{c\in C}\sum_{i\in I^R} PRR_c^t x^t_{iR^Rc} \\
&+ \sum_{p\in P}\sum_{i\in I^I}\sum_{j\in ER} PER^t_{jp} x^t_{ijp} + \sum_{p\in P}\sum_{i\in I^R} PSM^t_p x^t_{iSMp}\Bigg] \quad \text{Revenues} \\
&- \sum_{t\in T}\Bigg[\sum_{i\in I^I} FI_i^t (y_i^t - y_i^{t-1}) + \sum_{i\in I^R} FR_i^t (z_i^t - z_i^{t-1})\Bigg] \quad \text{Fixed (facilities)} \\
&- \sum_{t\in T}\Bigg[\sum_{i\in I^I}\sum_{q\in Q^I} FKI^t_{iq} u^t_{iq} + \sum_{i\in I^R}\sum_{q\in Q^R} FKR^t_{iq} v^t_{iq}\Bigg] \quad \text{Fixed (capacity modules)} \\
&- \sum_{t\in T}\sum_{p\in P}\Bigg[\sum_{i\in I^G}\sum_{j\in I^I} OI^t_{jp} x^t_{ijp} + \sum_{i\in I^R} OR^t_{ip} x^t_{iSMp}\Bigg] \quad \text{Operational} \\
&- \sum_{t\in T}\Bigg[\sum_{p\in P}\sum_{i\in I^G}\sum_{j\in I^I} T^t_{ijp} x^t_{ijp} + \sum_{c\in C}\sum_{i\in I^I}\sum_{j\in I^R} T^t_{ijc} x^t_{ijc}\Bigg] \quad \text{Transportation} \\
&- \sum_{t\in T}\sum_{c\in C}\sum_{i\in I^R} IC^t_{ic} I^t_{ic} \quad \text{Inventory} \\
&- \sum_{t\in T}\sum_{c\in C}\sum_{i\in I^R} BC^t_{ic} b^t_{ic}, \quad \text{Purchasing} \qquad (1)
\end{aligned}
$$

s.t.

Flow balance:

$$S^t_{ip} = x^t_{iR^Gp} + \sum_{j\in I^I} x^t_{ijp}, \quad i\in I^G,\ p\in P,\ t\in T, \qquad (2)$$

$$\sum_{j\in I^G} x^t_{jip} = \sum_{j\in ER} x^t_{ijp} + \frac{1}{\alpha_{pc}} x^t_{iR^Ic} + \sum_{j\in I^R} \frac{1}{\alpha_{pc}} x^t_{ijc}, \quad i\in I^I, p\in P, c\in C_p, t\in T, \qquad (3)$$

$$\sum_{j\in I^I} x^t_{jic} + I^{t-1}_{ic} + b^t_{ic} = x^t_{iR^Rc} + \alpha_{pc} x^t_{iSMp} + I^t_{ic}, \quad i\in I^R,\ p\in P,\ c\in C_p,\ t\in T, \qquad (4)$$

Demand:

$$\sum_{i\in I^R} x^t_{iSMp} \leqslant D^t_p, \quad p\in P,\ t\in T, \qquad (5)$$

Capacity:

$$\sum_{i\in I^I}\sum_{p\in P} x^t_{ijp} \leqslant KER^t_j, \quad j\in ER,\ t\in T, \qquad (6)$$

$$\sum_{j\in I^G}\sum_{p\in P} \gamma I_p x^t_{jip} \leqslant \sum_{\tau=1}^{t}\sum_{q\in Q^I} KI_q u^\tau_{iq}, \quad i\in I^I,\ t\in T, \qquad (7)$$

$$\sum_{p\in P} \gamma R_p x^t_{iSMp} \leqslant \sum_{\tau=1}^{t}\sum_{q\in Q^R} KP_q v^\tau_{iq}, \quad i\in I^R,\ t\in T, \qquad (8)$$

$$\sum_{j\in I^I}\sum_{c\in C} x^t_{jic} \leqslant \sum_{\tau=1}^{t}\sum_{q\in Q^R} KH_q v^\tau_{iq}, \quad i\in I^R,\ t\in T, \qquad (9)$$

$$\sum_{c\in C} \gamma_c I^t_{ic} \leqslant \sum_{\tau=1}^{t}\sum_{q\in Q^R} KIN_q v^\tau_{iq}, \quad i\in I^R,\ t\in T, \qquad (10)$$

Problem description

❖ Mathematical formulation (Continued)

$$\sum_{q \in Q^I} u_{iq}^t \leqslant y_i^t, \quad i \in I^I,\ t \in T, \tag{11}$$

$$\sum_{q \in Q^R} v_{iq}^t \leqslant z_i^t, \quad i \in I^R,\ t \in T, \tag{12}$$

Capacity expansion (one module)
(inspection/remanufacturing centers)

$$\sum_{j \in I^G} \sum_{p \in P} x_{jip}^t \geqslant MI_i^t y_i^t, \quad i \in I^I,\ t \in T, \tag{13}$$

$$\sum_{p \in P} x_{iSMp}^t \geqslant MR_i^t z_i^t, \quad i \in I^R,\ t \in T, \tag{14}$$

Minimum throughput
(inspection/remanufacturing centers)

$$y_i^t \leqslant y_i^{t+1}, \quad i \in I^I,\ t \in T \setminus \{|T|\}, \tag{15}$$

$$z_i^t \leqslant z_i^{t+1}, \quad i \in I^R,\ t \in T \setminus \{|T|\}, \tag{16}$$

Operating (end of the planning horizon)
(inspection/remanufacturing centers)

$$x_{ijp}^t \geqslant 0, \quad (i,j) \in A,\ p \in P, C,\ t \in T, \tag{17}$$

$$I_{ic}^t \geqslant 0, \quad i \in I^R,\ c \in C,\ t \in T, \tag{18}$$

$$b_{ic}^t \geqslant 0, \quad i \in I^R,\ c \in C,\ t \in T, \tag{19}$$

$$y_i^t \in \{0,1\}, \quad i \in I^I,\ t \in T, \tag{20}$$

$$z_i^t \in \{0,1\}, \quad i \in I^R,\ t \in T, \tag{21}$$

$$u_{iq}^t \in \{0,1\}, \quad i \in I^I,\ q \in Q^I,\ t \in T, \tag{22}$$

$$v_{iq}^t \in \{0,1\}, \quad i \in I^R,\ q \in Q^R,\ t \in T. \tag{23}$$

Condition

A case study

- Washing machines and tumble dryers in Germany

  - Washing machines and tumble dryers collected from 40 collection centers (cities).
  - All of the 40 cities are taken as potential sites for both inspection centers and remanufacturing facilities.
  - A 5-years planning horizon, two capacity modules (low and high), one extra remanufacturing facility with unlimited capacity.
  - Used the optimization software CPLEX 11.2.

A case study

❖ Computational results

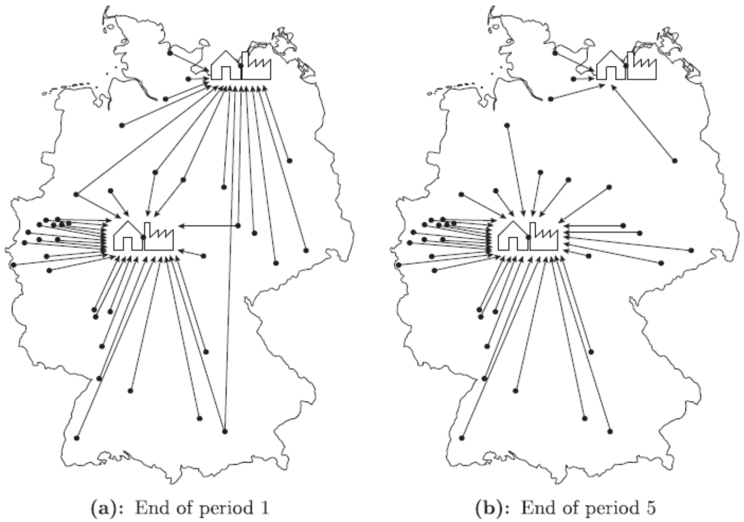

Figure 2. A solution of the problem

✓ Co-locating the inspection centers and remanufacturing facilities.

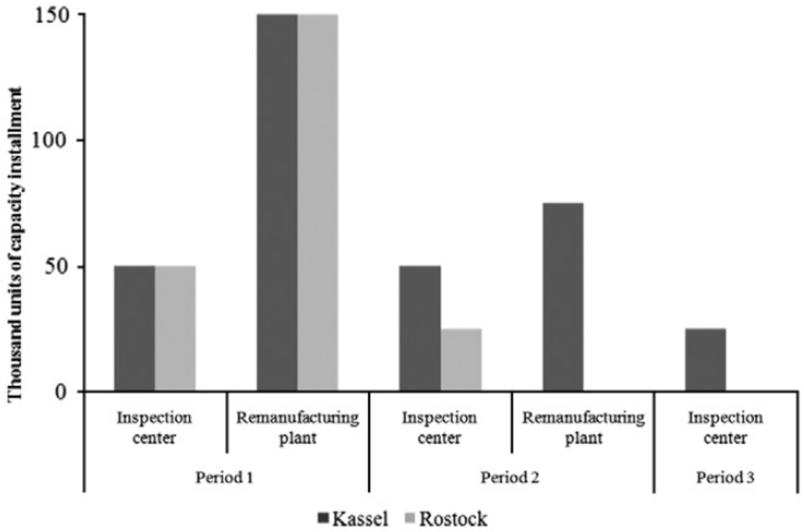

Figure 3. Capacity installment decisions in the solution

A case study

❖ Computational results (continued)

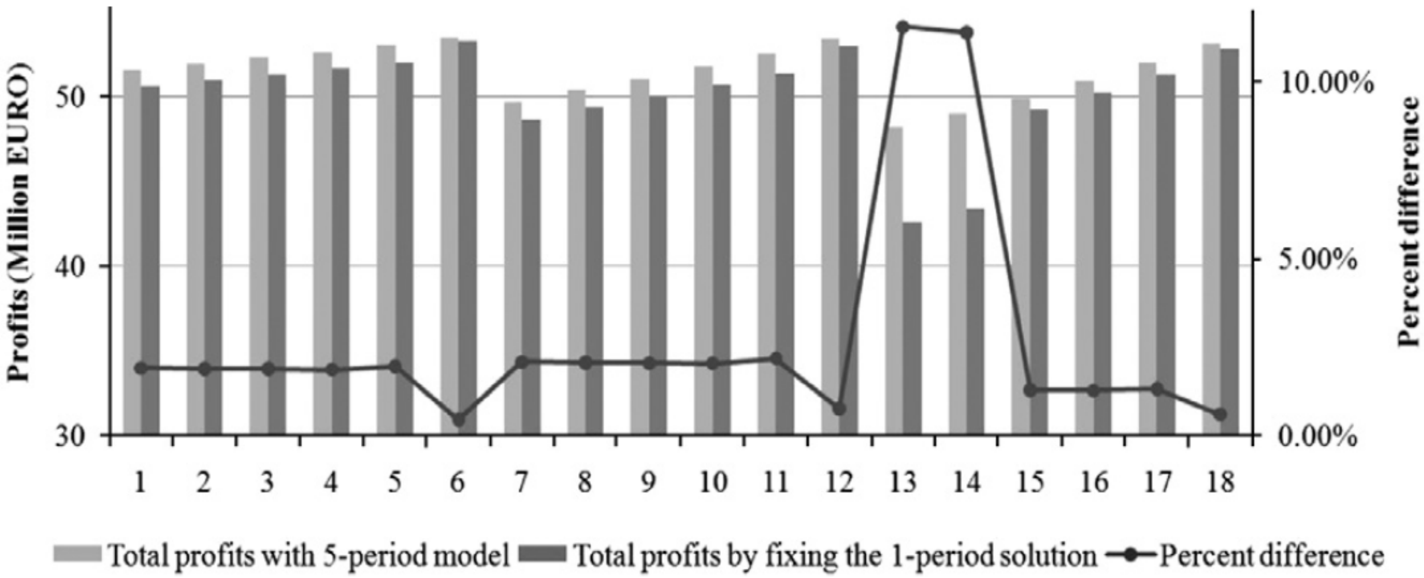

Figure 4. The value of the multi-period model over the 5-year planning horizon.

A case study

❖ Computational results (continued)

Table 2. The value of the multi-period model over the 5-year planning horizon.

Instance number	Total profits by using the 5-period model (EUR)	Total profits by fixing the 1-period solution (EUR)	Percent difference
1	51,584,835	50,600,344	1.91
2	51,923,433	50,938,942	1.90
3	52,262,032	51,277,540	1.88
4	52,600,630	51,616,138	1.87
5	52,992,312	51,954,737	1.96
6	53,479,368	53,245,925	0.44
7	49,663,337	48,628,470	2.08
8	50,352,614	49,311,692	2.07
9	51,041,892	49,994,913	2.05
10	51,735,096	50,678,134	2.04
11	52,507,776	51,361,356	2.18
12	53,362,789	52,957,426	0.76
13	48,184,468	42,613,639	11.56
14	48,982,028	43,398,169	11.40
15	49,856,320	49,208,082	1.30
16	50,890,221	50,241,983	1.27
17	51,963,557	51,275,884	1.32
18	53,122,824	52,802,854	0.60

Conclusion

❖ Summary

- Proposed a mathematical programming framework for multi-period reverse logistic network design problem.
- Proposed model accommodates several features of practical relevance.
- Utilizing the proposed model, instances with realistic sizes can be solved to optimality by using a commercial solver.

Thank You !

Q and A