An empirical comparison of Tabu Search, Simulated Annealing, and Genetic Algorithms for facilities location problems

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❖ Introduction

- Scope of research
  - Integrated capacity planning and warehouse location - forward logistics.
    - Amounts of capacity to be installed in the production plants in each period.
    - Were to locate and operate the warehouses in each period. (given set of plants producing for potential interconnected warehouses)

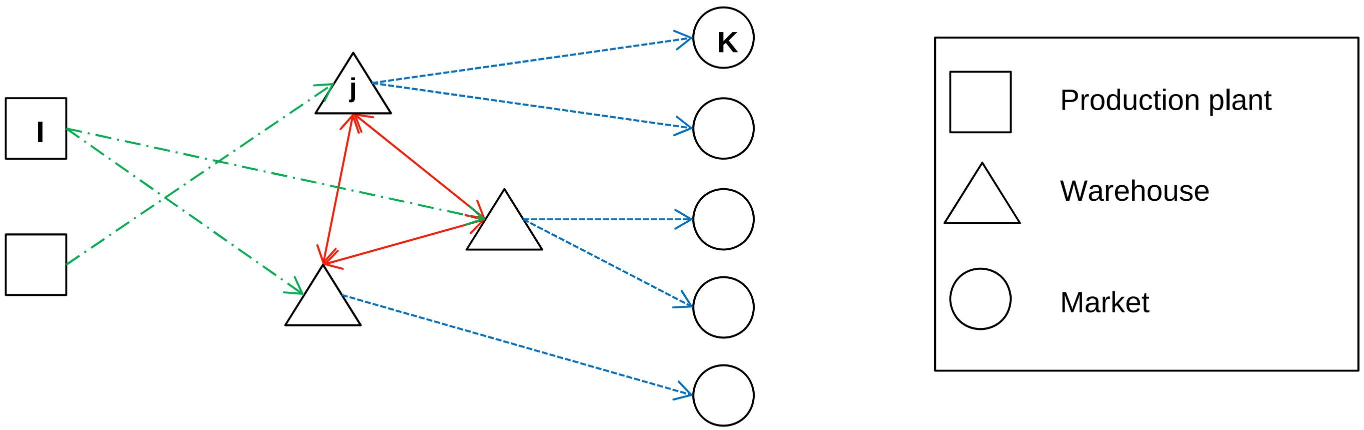

Figure 1. Network representation

Introduction

❖ Introduction

- Scope of research (continued)
  - Capacity planning and warehouse location problem with uncertain demands
    - Develop a deterministic capacity planning and warehouse location model.
    - Extend the deterministic capacity planning and warehouse location model to include uncertainty inherent in predicting volatile market demand.
    - Use the concept of robust optimization combined with Lagrangean relaxation method.

Introduction

❖ Literature review

- Related articles – forward logistics

Articles	Specifications
Cormier and Gunn (1999)	The pure capacity expansion problem
Rajagopalan et al. (1998) Lucas et al. (2001)	The capacity planning with uncertainty demands
Fong and Srinivasan (1981a, 1981b) Klincewicz et al. (1998) Lim and Kim (1999, 2001)	The integrated capacity planning and facility location

Problem description

- Capacity planning and warehouse location problem
  - Decision variables
    - Amounts of plant capacity, locations of warehouse
  - Objective
    - Minimizes total capacity investment and warehouse operating costs (total distribution and transfer costs)
  - Main constraints
    - Capacity, flow conservation, and market demand satisfaction constraints

Problem description

❖ Deterministic model

Model parameters

- α_i^t — discounted fixed cost for capacity expansion in plant i in period t
- β_i^t — discounted variable cost for capacity expansion in plant i in period t
- ψ_j^t — fixed cost of operating warehouse j during period t
- $\varphi_{jj'}^t$ — transfer cost from warehouse j to warehouse j' at the end of period t when $j \neq j'$, and inventory holding cost in warehouse j at the end of period t when $j = j'$
- γ_{ij}^t — shipment cost from plant i to warehouse j during period t
- δ_{jk}^t — shipment cost from warehouse j to market k during period t
- υ_i^0 — initial capacity in plant i at the beginning of the planning horizon
- m_i^t — the maximum capacity expansion in plant i during period t
- k_j^t — capacity of warehouse j in period t
- d_k^t — demand of market k in period t (assumed to be stochastic and discrete)

Model variables

- x_i^t — binary decision variable set to 1 if the capacity in plant i is to be expanded during period t, and 0 otherwise
- u_i^t — capacity expansion level for plant i in period t
- y_j^t — binary decision variable set to 1 if warehouse j is to be operated during period t, and 0 otherwise
- v_{ij}^t — proportion of the available capacity in plant i assigned to warehouse j in period t
- w_{jk}^t — proportion of market k demand d_k^t that is satisfied from the warehouse j in period t
- $f_{jj'}^t$ — amount transferred from warehouse j to warehouse j' at the end of period t

Problem description

❖ Deterministic model (continued)

$$\text{Minimize} \;\; Z_{IP} = \sum_{t\in T}\sum_{i\in I}\left(\alpha_i^t x_i^t + \beta_i^t u_i^t + \sum_{j\in J}\gamma_{ij}^t v_{ij}^t\right)$$

Plant costs
(fixed, variable, and transportation costs)

$$+ \sum_{t\in T}\sum_{j\in J}\left(\psi_j^t y_j^t + \sum_{j'\in J}\varphi_{jj'}^t f_{jj'}^t + \sum_{k\in k}\delta_{jk}^t w_{jk}^t\right)$$

warehouse costs
(fixed, variable, and transportation costs)

subject to

$$\sum_{j\in J} v_{ij}^t - \sum_{\tau=1}^{t} u_i^\tau \leqslant v_i^0 \quad \forall i, t \qquad (1)$$

$$u_i^t - m_i^t x_i^t \leqslant 0 \quad \forall i, t \qquad (2)$$

Capacity constraints

$$\sum_{i\in I} v_{ij}^t + \sum_{j'\in J} f_{j'j}^{t-1} - \sum_{k\in K} w_{jk}^t - \sum_{j'\in J} f_{jj'}^t = 0 \quad \forall j, t \qquad (3)$$

$$\sum_{k\in K} w_{jk}^t + \sum_{j'\in J} f_{jj'}^t - \kappa_j^t y_j^t \leqslant 0 \quad \forall j, t \qquad (4)$$

Flow conservation constraints

$$\sum_{k\in K} w_{jk}^t = d_k^t \quad \forall k, t \qquad (5)$$

Market demand satisfaction constraint

$$u_i^t, v_{ij}^t, f_{jj'}^t, w_{jk}^t \geqslant 0, \quad x_i^t, y_j^t \in \{0, 1\} \quad \forall i, j, j', k, t$$

Problem description

$PS = \{1, 2, \ldots, S\}$ ← Future possible scenarios

$\sum_{s \in PS} p_s = 1$ ← Probability of occurrence

$\{E_s, F_s, d_s, g_s\}$ ← Control constraints

❖ A robust optimization model

- Robust optimization paradigm

(LP):

$$
\begin{aligned}
\text{Minimize} \quad & cx + dy \\
\text{Subject to} \quad & Ax = b \qquad (6) \\
& Ex + Fy = g \qquad (7) \\
& x, y \geqslant 0
\end{aligned}
$$

x : design variables (independent of uncertainty parameters)
y : control variables (adjusted following uncertainty parameters)
c, d, A, b, E, F, g : input parameters

There is no guarantee that the control constraints will always be satisfied. (error variables)

$$
\begin{aligned}
\text{Minimize} \quad & \sigma(x, y_1, y_2, \ldots, y_s) + \omega\rho(e_1, e_2, \ldots, e_s) \\
\text{Subject to} \quad & Ax = b \qquad (6) \\
& E_s x + F_s y + e_s = g_s \qquad (8) \\
& x, y_s \geqslant 0
\end{aligned}
$$

(6) ← Design constraint

(8) ← Control constraint

Problem description

❖ A robust optimization model (continued)

$(\mathrm{CPWL_R})$:

$$\text{Minimize} \quad Z_R = \sum_{s \in PS} p_s \left(\xi_s + \omega_1 \sum_{t \in T} \sum_{i \in I} e_i^{ts} + \omega_2 \sum_{t \in T} \sum_{j \in J} \varepsilon_j^{ts} \right)$$

subject to

$$\sum_{j \in J} v_{ij}^{ts} - \sum_{\tau=1}^{t} u_i^{\tau} - e_i^{ts} \leqslant \upsilon_i^0 \quad \forall i, t, s \qquad (9)$$

$$u_i^t - m_i^t x_i^t \leqslant 0 \quad \forall i, t \qquad (2)$$

$$\sum_{i \in I} v_{ij}^{ts} + \sum_{j' \in J} f_{j'j}^{t-1.s} - \sum_{k \in K} w_{jk}^{ts} - \sum_{j' \in J} f_{jj'}^{ts} = 0 \quad \forall j, t, s \qquad (10)$$

$$\sum_{k \in K} w_{jk}^{ts} + \sum_{j' \in J} f_{jj'}^{ts} - \kappa_j^t y_j^t - \varepsilon_j^{ts} \leqslant 0 \quad \forall j, t, s \qquad (11)$$

$$\sum_{k \in K} w_{jk}^{ts} = d_k^{ts} \quad \forall j, t, s \qquad (12)$$

$$u_i^t, v_{ij}^{ts}, f_{jj'}^{ts}, w_{jk}^{ts}, e_i^{ts}, \varepsilon_j^{ts} \geqslant 0, \; x_i^t, y_j^t \in \{0, 1\} \quad \forall i, j, j', k, t, s$$

$$\xi_s = \sum_{t \in T} \sum_{i \in I} \left(\alpha_i^t x_i^t + \beta_i^t u_i^t + \sum_{j \in J} \gamma_{ij}^t v_{ij}^{ts} \right) + \sum_{t \in T} \sum_{j \in J} \left(\psi_j^t y_j^t + \sum_{j' \in J} \varphi_{jj'}^t f_{jj'}^{ts} + \sum_{k \in k} \delta_{jk}^t w_{jk}^{ts} \right)$$

- Capacity planning and warehouse location decision variables (for the entire planning horizon) are the design variable
- Distribution decisions are control variables

Problem description

❖ Reformulate optimization model for decomposition

(CPWL$^{\mathrm{R}}_{\mathrm{R}}$):

$$\text{Minimize} \quad Z_R = \sum_{s \in PS} p_s \left(\xi'_s + \omega_1 \sum_{t \in T} \sum_{i \in I} e_i^{ts} + \omega_2 \sum_{t \in T} \sum_{j \in J} \varepsilon_j^{ts} \right)$$

subject to

$$\sum_{j \in J} v_{ij}^{ts} - \sum_{\tau=1}^{t} u_i^{\tau s} - e_i^{ts} \leqslant v_i^0 \quad \forall i, t, s \quad (9)$$

$$u_i^{ts} - m_i^t x_i^{ts} \leqslant 0 \quad \forall i, t, s \quad (13)$$

$$\sum_{i \in I} v_{ij}^{ts} + \sum_{j' \in J} f_{j'j}^{t-1,s} - \sum_{k \in K} w_{jk}^{ts} - \sum_{j' \in J} f_{jj'}^{ts} = 0 \quad \forall j, t, s \quad (10)$$

$$\sum_{k \in K} w_{jk}^{ts} + \sum_{j' \in J} f_{jj'}^{ts} - \kappa_j^t y_j^{ts} - \varepsilon_j^{ts} \leqslant 0 \quad \forall j, t, s \quad (14)$$

$$\sum_{k \in K} w_{jk}^{ts} = d_k^{ts} \quad \forall j, t, s \quad (12)$$

$$u_i^t - m_i^t x_i^t \leqslant 0 \quad \forall i, t \quad (2)$$

$$x_i^{ts} - x_i^t = 0 \quad \forall i, t, s \quad (15)$$

$$\xi'_s = \sum_{t \in T} \sum_{i \in I} \left(\alpha_i^t x_i^{ts} + \beta_i^t u_i^{ts} + \sum_{j \in J} \gamma_{ij}^t v_{ij}^{ts} \right) + \sum_{t \in T} \sum_{j \in J} \left(\psi_j^t y_j^{ts} + \sum_{j' \in J} \varphi_{jj'}^t f_{jj'}^{ts} + \sum_{k \in k} \delta_{jk}^t w_{jk}^{ts} \right)$$

$$u_i^{ts} - u_i^t = 0 \quad \forall i, t, s \quad (16)$$

$$y_j^{ts} - y_j^t = 0 \quad \forall j, t, s \quad (17)$$

$$u_i^t,\ u_i^{ts},\ v_{ij}^{ts},\ f_{jj'}^{ts},\ w_{jk}^{ts} \geqslant 0,\ x_i^t,\ x_i^{ts},\ y_j^t,\ y_j^{ts} \in \{0, 1\} \quad \forall i, j, j', k, t, s$$

- Capacities are decide → distribution plans are decide
- x, u, and y, and constraints (2) are the only components linking the solutions of the capacity planning and warehouse location problem
- Introduce new variables x_i^{ts}, u_i^{ts}, and y_j^{ts} defining, respectively

Problem description

❖ Decomposition models

(CPWL$_{\mathrm{R}}^{\mathrm{LR}}$(s)):

$$\text{Minimize} \quad Z^{LR}(s) = \xi_s'' + p_s \sum_{t\in T}\sum_{i\in I} \omega_1 e_i^{ts} + p_s \sum_{t\in T}\sum_{j\in J} \omega_2 \varepsilon_j^{ts}$$

subject to

$$\sum_{j\in J} v_{ij}^{ts} - \sum_{\tau=1}^{t} u_i^{\tau s} - e_i^{ts} \leqslant \upsilon_i^0 \quad \forall i,t \quad (9)$$

$$u_i^{ts} - m_i^t x_i^{ts} \leqslant 0 \quad \forall i,t \quad (13)$$

$$\sum_{i\in I} v_{ij}^{ts} + \sum_{j'\in J} f_{j'j}^{t-1,s} - \sum_{k\in K} w_{jk}^{ts} - \sum_{j'\in J} f_{jj'}^{ts} = 0 \quad \forall j,t \quad (10)$$

$$\sum_{k\in K} w_{jk}^{ts} + \sum_{j'\in J} f_{jj'}^{ts} - \kappa_j^t y_j^{ts} - \varepsilon_j^{ts} \leqslant 0 \quad \forall j,t \quad (14)$$

$$\sum_{k\in K} w_{jk}^{ts} = d_k^{ts} \quad \forall j,t \quad (12)$$

$$u_i^{ts},\ v_{ij}^{ts},\ f_{jj'}^{ts},\ w_{jk}^{ts} \geqslant 0,\ x_i^{ts},\ y_j^{ts} \in \{0,1\} \quad \forall i,j,j',k,t$$

$$\xi_s'' = \sum_{t\in T}\sum_{i\in I}\Bigg((p_s\alpha_i^t - \lambda_i^{ts})x_i^{ts} + (p_s\beta_i^t - \mu_i^{ts})u_i^{ts} + p_s \sum_{j\in J} \gamma_{ij}^t v_{ij}^{ts} \Bigg) + \sum_{t\in T}\sum_{j\in J}\Bigg((p_s\psi_j^t - \eta_j^{ts})y_j^{ts} + p_s \sum_{j'\in J} \varphi_{jj'}^t f_{jj'}^{ts} + p_s \sum_{k\in k} \delta_{jk}^t w_{jk}^{ts} \Bigg)$$

$$\text{Minimize} \quad Z^{\mathrm{LR}}(0) = \sum_{t\in T}\sum_{i\in I}\left(\left(\sum_{s\in PS} \lambda_i^{ts}\right) x_i^t + \left(\sum_{s\in PS} \mu_i^{ts}\right) u_i^t \right) + \sum_{t\in T}\sum_{j\in J}\left(\sum_{s\in PS} \eta_j^{ts}\right) y_j^t$$

subject to

$$u_i^t - m_i^t x_i^t \leqslant 0 \quad \forall i,t \quad (2)$$

$$u_i^t \geqslant 0,\ x_i^t,\ y_j^t \in \{0,1\} \quad \forall i,j,t$$

Solution algorithm

❖ Decomposition based approximation algorithm

Step 0

All decision variables are set to 1

Step 1

$Z^{\mathrm{LR}}(s)$

Relaxation problem is solved

Step 2

$Z^{\mathrm{LR}}(0)$

Common capacity expansion and warehouse location plans are solved

Terminate condition: repeated until no significant change in the total cost

$$\sum_{s\in PS} p_s Z^{\mathrm{LR}}(s) + Z^{\mathrm{LR}}(0)$$

$(\mathrm{CPWL}_{\mathrm{R}}^{\mathrm{LR}}\ (\mathrm{s})_{\mathrm{LP}})$:

$$\text{Minimize } Z_{\mathrm{LP}}^{\mathrm{LR}}(s) = \xi_s' + p_s \sum_{t\in T}\sum_{i\in I} \omega_1 e_i^{ts} + p_s \sum_{t\in T}\sum_{j\in J} \omega_2 \varepsilon_j^{ts}$$

subject to

$$\sum_{j\in J} v_{ij}^{ts} - \sum_{\tau=1}^{t} u_i^{\tau s} - e_i^{ts} \leqslant \upsilon_i^0 \quad \forall i,t \quad (9)$$

$$\sum_{k\in K} w_{jk}^{ts} = d_k^{ts} \quad \forall j,t \quad (12)$$

$$u_i^{ts} - m_i^t x_i^{ts} \leqslant 0 \quad \forall i,t \quad (13)$$

$$x_i^{ts} - \hat{x}_i^t = 0 \quad \forall i,t \quad (15)$$

$$u_i^{ts} - \hat{u}_i^t = 0 \quad \forall i,t \quad (16)$$

$$\sum_{i\in I} v_{ij}^{ts} + \sum_{j'\in J} f_{j'j}^{t-1,s} - \sum_{k\in K} w_{jk}^{ts} - \sum_{j'\in J} f_{jj'}^{ts} = 0 \quad \forall j,t \quad (10)$$

$$y_j^{ts} - \hat{y}_j^t = 0 \quad \forall j,t \quad (17)$$

$$\sum_{k\in K} w_{jk}^{ts} + \sum_{j'\in J} f_{jj'}^{ts} - \kappa_j^t y_j^{ts} - \varepsilon_j^{ts} \leqslant 0 \quad \forall j,t \quad (14)$$

$$u_i^{ts},\ v_{ij}^{ts},\ f_{jj'}^{ts},\ w_{jk}^{ts} \geqslant 0,\ 0 \leqslant x_i^{ts} \leqslant 1,\quad 0 \leqslant y_j^{ts} \leqslant 1 \quad \forall i,j,j',k,t$$

Computational experiments

- Test results

  - Example
    - Two facility producing the same product, three potential warehouses, five market and five periods (results are given in Table 6)

  - Large scale-computational experiments (6 periods)
    - Consider small, medium, and large problem sizes
    - Considered small, medium, and large number of scenarios (25, 50, and 75)
    - Considered low, medium, and large demand variability (25, 50, and 75%),
    - Five problems are generated and solved with three approaches ($CPWL_R$, $CPWL_D$* and DEC-Alg)

Conclusion

❖ Summary

- Studied the capacity planning and warehouse location problem with uncertain demand.
  - Proposed a deterministic model when demands are known with certainty
  - Proposed a case of uncertain demand
- Developed an algorithm based on the Lagrangean relaxation decomposition.
- Decomposition-based algorithm produces rather good solution.

Thank You !

Q and A